15-251: Great Theoretical Ideas In Computer Science

Recitation 2

Announcements

- Be sure to start early on homeworks! It will be a smoother experience for all involved (less crowded office hours are better for you and us!)
- Please practice writing up your solutions beforehand. It will help with:
  - Structuring your proof on the page in an organized way.
  - Writing neatly (minimizing erasing/crossing out with practice).
  - Ensuring your solution is correct (often bugs pop up while writing).
- If you aren't sure about a piece of feedback or why you got a particular score on a problem, please ask us! The TAs initialed which problems they graded, so go to the correct TA if you are at all unsure. We're here to help!
- If you have any questions about the course overall, not restricted to homework, please ask! Again, we want to help you however we can!
- Reminder: Make sure that you're familiar with the collaboration policies on the website. The homework writing sessions and open collaboration problems are very different from many classes, so just make sure you're clear on the specifics.

Nature of Language

Which of the following languages are regular? Prove your answer.

(a) $L = \{xwx^R : x, w \in \Sigma^*, |x|, |w| > 0\}$, where $\Sigma = \{a, b\}$ and x^R means the reversal of x.

(b) $L = \{x : x = x^R, x \in \Sigma^*\}$, where $\Sigma = \{a, b\}$.

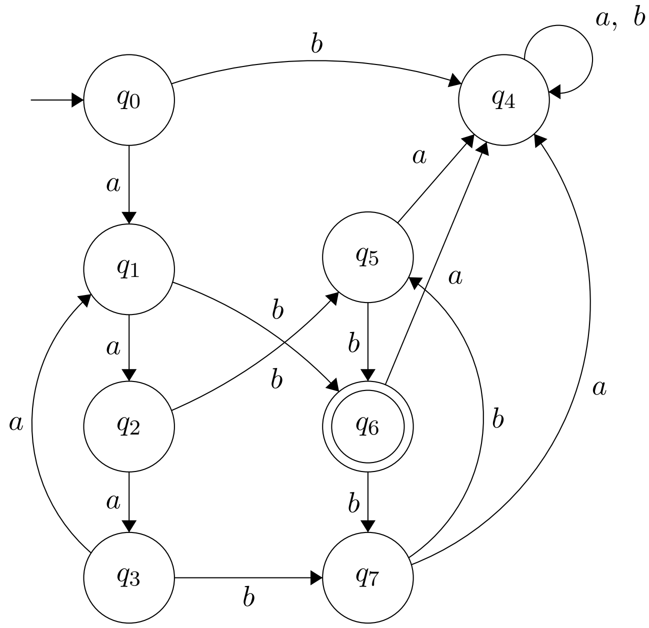
Closure of Regularity

(a) Prove that if L_1 and L_2 are regular, then $L_1 \cap L_2$ is regular.

(b) Using (a), show that $L = \{w : w \text{ has the same number of 0s and 1s}\}$ over the alphabet $\Sigma = \{0, 1\}$ is irregular.

DFAs Can Count

What language does the following DFA decide?



Tough Decisions

For each language below, draw a TM that decides the language. As usual, you can use any finite tape alphabet Γ containing Σ and $\sqcup$; just be sure to specify what it is. In addition, explain briefly in prose how the TM works, and the "meaning" of each state.

(a) $L = \{a^n : n \text{ is a nonnegative integer power of } 2\}$, where $\Sigma = \{a\}$.

(b) $L = \{x : x \text{ has the same number of } a\text{'s and } b\text{'s}\}$, where $\Sigma = \{a, b\}$.

Powerful Vocabulary

For a language $L \subseteq \{0,1\}^*$, define $\mathrm{perm}(L)$ to be the set of all permutations of all the words in L. Show that if L is decidable, then $\mathrm{perm}(L)$ is decidable.