15-251: Great Theoretical Ideas In Computer Science

Recitation 2

Training Manual

- **Deterministic Finite Automaton (DFA)**: A DFA M is a machine that reads a finite input one character at a time in one pass, transition from state to state, and ultimately accepts or rejects. Formally, M is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where Q is the finite set of states, Σ is the finite alphabet, $\delta : Q \times \Sigma \to Q$ is the transition function, $q_0 \in Q$ is the starting state, and $F \subseteq Q$ is the set of accepting states.
- **Regular language**: A language L is regular if $L = L(M)$ for some DFA M (M decides L).
- **Turing Machine (TM)**: A TM M is a machine that can read and write to an infinite tape containing the input, transition from state to state, and ultimately accepts or rejects. Formally, M is a 7-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$, where:
  - Q is the finite set of states,
  - Σ is the finite input alphabet with $\sqcup \notin \Sigma$,
  - Γ is the finite tape alphabet with $\sqcup \in \Gamma$ and $\Sigma \subset \Gamma$,
  - $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function,
  - $q_0 \in Q$ is the starting state,
  - $q_{acc} \in Q$ is the accepting state,
  - and $q_{rej} \in Q$ is the rejecting state.
- **Decider TM**: A TM M is a decider if it halts on all inputs.
- **Decidable language**: A language L is decidable (or computable) if $L = L(M)$ for some decider TM M.

Odd Ones Out

Draw a DFA that decides the language

$$L = \{x : x \text{ has an even number of 1s and an odd number of 0s}\}$$

over the alphabet $\Sigma = \{0, 1\}$.

Ones Too Many

Show that the language $L = \{1^n | \log_2(n) \in \mathbb{N}\}$ over the alphabet $\Sigma = \{1\}$ is not regular.

Busy Intersection

(a) Prove that if L_1 and L_2 are regular, then $L_1 \cap L_2$ is regular.

(b) Using (a), show that $L = \{w : w \text{ has the same number of 0s and 1s}\}$ over the alphabet $\Sigma = \{0, 1\}$ is not regular.

Balance in All Things

Construct a TM that decides the language $L = \{x : \text{the parentheses in } x \text{ are balanced}\}$ over the alphabet $\Sigma = \{(,)\}$.

Closure Ceremony

Suppose that L_1 and L_2 are decidable languages. Show that the three languages $L_1 \cup L_2$, $L_1 \cdot L_2$ and L_1^* are all decidable as well.