15-251: Great Theoretical Ideas In Computer Science

Recitation 5

Announcements

- Midterm next **Wednesday, October 5**! It will be held in **DH 2315** from **6.30pm** to 9.30pm in place of the writing session.
- Practice problems have been posted. We will be holding two **solution sessions** to go over these problems in place of the **Monday and Tuesday 6.30pm-8pm** office hours at **PH A18B (Monday)** and **SH 214 (Tuesday)**.
- All other office hours will continue as per normal. Make good use of them!
- We will also be hosting reviews at **GHC 4102** for the following topics:
  - Regularity: Saturday 2pm - 3pm
  - Countability: Saturday 5.30pm - 6.30pm
  - Decidability: Sunday 3pm - 4pm
- Small group sessions for this week have been moved from Thursday and Friday to Saturday and Sunday.
- Good luck for the midterm!

Definitions

- A **tree** is a connected acyclic graph.
- A **leaf** is a vertex in a tree with degree 1 (has exactly one neighbor).
- Given a connected weighted graph, a **minimum spanning tree** is a tree containing all the vertices of the graph of minimum total weight.
- A **bipartite graph** is a graph in which the vertices can be split into two bipartitions such that there is no edge between any two vertices in a single bipartition.
- A **matching** is a subset of the edges of a graph that do not share endpoints.

Primitive Problems

This question is about the Minimum Spanning Tree problem.

(a) Suppose an instance of the Minimum Spanning Tree problem is allowed to have negative costs for the edges. Explain whether the Jarník-Prim algorithm would work in this case as well.

(b) Consider the problem of computing the maximum spanning tree, i.e., a spanning tree that maximizes the sum of the edge costs. Explain whether the Jarník-Prim algorithm solves this problem if we modify it so that at each iteration, the algorithm chooses the edge between V' and $V \backslash V'$ with the maximum cost.

Maximum Matching

Let $G = (X, Y, E)$ be a bipartite graph. Give a polynomial-time algorithm that outputs a maximum matching in M.

Tree Trivia

(a) Show that every tree with $n \geq 2$ contains a leaf.

(b) Prove that every tree has at most one perfect matching.