

HOMEWORK 7
DUE NOV 10 BY EMAIL

1. In order help us reason about communication protocols, we will assume that the players always alternate sending bits. So first Alice sends a bit to Bob, then Bob sends a bit to Alice, then Alice sends a bit to Bob, etc. We assume at the end, the last bit communicated is the output of the protocol, which is supposed to be the same as the output of the function being computed. When we enforce the players to alternate sending bits, we may increase the cost of the protocol by a factor of at most 2.¹ But we don't care about constant factors.

Prove that $\mathbf{D}(\text{EQ}) \geq \Omega(n)$, where EQ denotes the equality function seen in class.

Hint: Take the best protocol computing EQ, and suppose it has cost c . Let $\Pi(x, y)$ denote the sequence of bits communicated by the protocol when the input is (x, y) . Note that for all $(x, y) \in \{0, 1\}^n \times \{0, 1\}^n$, $|\Pi(x, y)| \leq c$. Is it possible that $\Pi(x, x) = \Pi(y, y)$ for $x \neq y$?

2. Let $L = \{ww : w \in \{0, 1\}^*\}$. Prove that L is not regular by arguing that if it was regular, then $\mathbf{D}(\text{EQ}) \leq O(1)$ (which contradicts Question 1).

¹So when this restriction is enforced, we cannot claim that $\mathbf{D}(F) \leq n + 1$ for every $F : \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}$, but we can say $\mathbf{D}(F) \leq 2n + 1$.