New Phrases

- We say a language is in $P$ if there exists a polynomial time algorithm that decides the language.
- We say a problem is in $NP$ if there exists a polynomial time verifier $TM$ $V$ such that for all $x \in \Sigma^*$, $x$ is in $L$ if and only if there exists a polynomial length certificate $u$ such that $V(x, u) = 1$.
- We say there is a polynomial-time many-one reduction from $A$ to $B$ if there is a polynomial-time computable function $f : \Sigma^* \rightarrow \Sigma^*$ such that $x \in A$ if and only if $f(x) \in B$. We write this as $A \leq^P_m B$. (We also refer to these reductions as Karp reductions.)
- A problem $Y$ is $NP$-hard if for every problem $X \in NP$, $X \leq^P_m Y$.
- A problem is $NP$-complete if it is both in $NP$ and $NP$-hard.

Not oPen

Show that $NP$ is closed under union and intersection. Specifically, prove that if two languages, $L_1, L_2 \in NP$, then $L_1 \cap L_2 \in NP$ and $L_1 \cup L_2 \in NP$.

No Peeking

We define a vertex covering of a graph as a set of vertices such that each edge in the graph is incident to at least one vertex in the set.

$\text{VERTEX-COVER}: \{(G, k) : G \text{ is a graph, } k \text{ a natural number, } G \text{ contain a vertex covering of size } k\}$

Show $\text{VERTEX-COVER}$ is $NP$-complete (Try reducing from 3SAT).

(Extra) No Privacy

$\text{DOUBLE-CLIQUE}$: Given a graph $G = (V, E)$ and a natural number $k$, does $G$ contain two vertex-disjoint cliques of size $k$ each?

Show $\text{DOUBLE-CLIQUE}$ is $NP$-Complete.

(Bonus) Never Pausing

Prove that the Halting Problem is $NP$-hard.