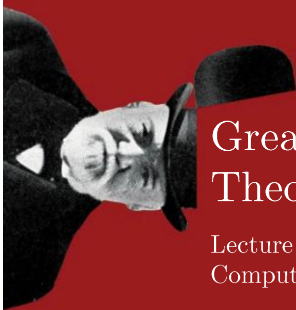

Great Ideas in Theoretical CS

Lecture 19: Computational Social Choice

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Social choice theory

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Arrow and Sen

The voting model

- Set of voters $N = \{1, \dots, n\}$
- Set of alternatives A; denote $|A| = m$
- Each voter has a ranking over the alternatives
- Preference profile = collection of all voters' rankings

1	2	3
a	*c*	*b*
b	*a*	*c*
c	*b*	*a*

Vote over cuisines

Indian	Japanese	Chinese	Italian	Mexican
(In)	(J)	(C)	(It)	(M)

Voting rules

- Voting rule = function from preference profiles to alternatives that specifies the winner of the election
- Plurality
  - Each voter awards one point to top alternative
  - Alternative with most points wins
  - Used in almost all political elections

More voting rules

- Borda count
  - Each voter awards $m - k$ points to alternative ranked k'th
  - Alternative with most points wins
  - Proposed in the 18th Century by the chevalier de Borda
  - Used for national elections in Slovenia
  - Similar to rule used in the Eurovision song contest

Lordi
Eurovision 2006 winners

More voting rules

- x beats y in a pairwise election if the majority of voters prefer x to y
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

More voting rules

- Single Transferable vote (STV)
  - $m - 1$ rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in:
    - Ireland, Malta, Australia, and New Zealand
    - US: Maine (governor, US congress), cities like San Francisco and Cambridge

STV: example

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

2 voters	2 voters	1 voter
b	b	b

Social choice axioms

- How do we choose among the different voting rules? Via desirable properties!
- Majority consistency = if a majority of voters rank alternative x first, then x should be the winner
- Poll 1: Which rule is not majority consistent?
  1. Plurality
  2. Plurality with runoff
  3. Borda count
  4. STV

Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison

Condorcet winner

- Recall: x beats y in a pairwise election if a majority of voters rank x above y
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox = cycle in majority preferences

1	2	3
a	c	b
b	a	c
c	b	a

Condorcet consistency

- Condorcet consistency = select a Condorcet winner if one exists
- Poll 2: Which rule is Condorcet consistent?
  1. Plurality
  2. Borda count
  3. Both
  4. Neither

More voting rules

- Copeland
  - Alternative's score is #alternatives it beats in pairwise elections
  - Why does Copeland satisfy the Condorcet criterion?

Awesome example

- Plurality: a
- Borda: b
- Condorcet winner: c
- STV: d
- Plurality with runoff: e

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Manipulation

- Using Borda count
- Top profile: b wins
- Bottom profile: a wins
- By changing his vote, voter 3 achieves a better outcome!
- Borda's response: "My scheme is intended only for honest men!"

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Strategyproofness

- A voting rule is strategyproof (SP) if a voter can never benefit from lying about his preferences
- Poll 3: What is the largest value of m for which plurality is SP?
  1. $m = 1$
  2. $m = 2$
  3. $m = 3$
  4. $m = \infty$

Strategyproofness

- A voting rule is dictatorial if there is a voter who always gets his most preferred alternative
- A voting rule is constant if the same alternative is always chosen
- Constant functions and dictatorships are SP


Dictatorship


Constant function

Gibbard-Satterthwaite

- A voting rule is onto if any alternative can win
- Theorem (Gibbard-Satterthwaite): If $m \geq 3$ then any voting rule that is SP and onto is dictatorial
- In other words, any voting rule that is onto and nondictatorial is manipulable


Gibbard


Satterthwaite

Complexity of manipulation

- Manipulation is always possible in theory
- But can we design voting rules where it is difficult in practice?
- Are there “reasonable” voting rules where manipulation is a hard computational problem? [Bartholdi et al. 1989]

The computational problem

- f-Manipulation problem:
  - Given votes of nonmanipulators and a preferred alternative p
  - Can manipulator cast vote that makes p uniquely win under f?
- Example: Borda, $p = a$

1	2	3
b	b	
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

A greedy algorithm

- Rank p in first place
- While there are unranked alternatives:
  - If there is an alternative that can be placed in next spot without preventing p from winning, place this alternative
  - Otherwise return false

Example: Borda

1	2	3
b	b	a
a	a	
c	c	
d	d	

1	2	3
b	b	a
a	a	b
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	
d	d	

1	2	3
b	b	a
a	a	c
c	c	b
d	d	

1	2	3
b	b	a
a	a	c
c	c	d
d	d	

1	2	3
b	b	a
a	a	c
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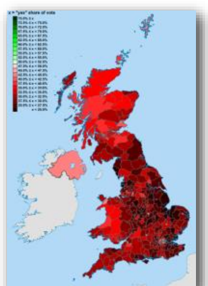
When does the alg work?

- Fact: The greedy algorithm is a polynomial-time algorithm for R-Manipulation for $R \in$ {plurality, Borda count, plurality with runoff, Copeland,...}
- Theorem [Bartholdi and Orlin, 1991]: The STV-Manipulation problem is NP-complete!

Is social choice practical?

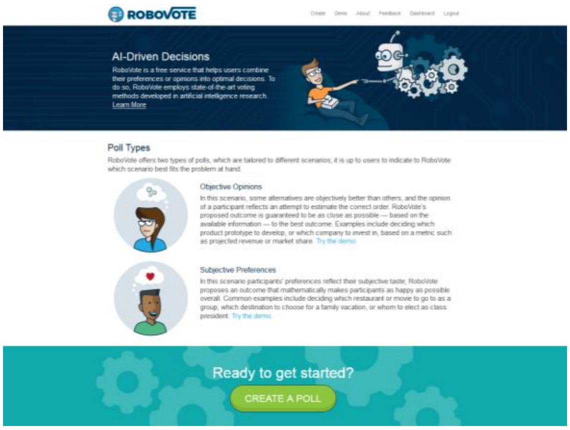
- UK referendum: Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better...
- ... but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



Computational social choice

- However:
  - in online voting...
  - in human computation...
  - in multiagent systems...

  the designer is free to employ any voting rule!



SUMMARY

- Terminology:
  - Plurality, Borda count, plurality with runoff, STV, Copeland
  - Majority consistency
  - Condorcet winner, Condorcet consistency
  - Strategyproofness
  - The Gibbard-Satterthwaite Thm
- Principles:
  - NP-hardness can be good!

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