These Decidable Definitions Have Undecidable Ends

- A **decider** is a TM that halts on all inputs.
- A language $L$ is **undecidable** if there is no TM $M$ that halts on all inputs such that $M(x)$ accepts if and only if $x \in L$.
- A language $A$ **reduces** to $B$ if it is possible to decide $A$ using an algorithm that decides $B$ as a subroutine. Denote this as $A \leq B$ (read: $B$ is at least as hard as $A$)

Doesn’t Look Like Anything (Decidable) To Me

Prove that the following languages are undecidable (below, $M$, $M_1$, $M_2$ refer to TMs).

(a) $\text{REGULAR} = \{\langle M \rangle : L(M) \text{ is regular} \}$.

(b) $\text{TOTAL} = \{\langle M \rangle : M \text{ halts on all inputs} \}$.

(c) $\text{DOLORES} = \{\langle M_1, M_2 \rangle : \exists w \in \Sigma^* \text{ such that both } M_1(w) \text{ and } M_2(w) \text{ accept} \}$.

(Extra) Lose All Scripted Responses. Improvisation Only

Let $\text{FINITE} = \{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is finite} \}$.

Show that $\text{TOTAL} \leq \text{FINITE}$.