15-251: Great Theoretical Ideas In Computer Science

Recitation 4

Announcements

Reminders :

- Midterm 1 Solution Sessions - Saturday 1:30-2:30 in GHC 4301 and Sunday 1:30-2:30 in GHC 4215
- Remember to submit answers to the weekly quiz by 9 pm Sunday.

These Decidable Definitions Have Undecidable Ends

- A **decider** is a TM that halts on all inputs.
- A language L is **undecidable** if there is no TM M that halts on all inputs such that $M(x)$ accepts if and only if $x \in L$.
- A language A **reduces** to B if it is possible to decide A using an algorithm that decides B as a subroutine. Denote this as $A \leq B$ (read: B is *at least* as hard as A)

Doesn't Look Like Anything (Decidable) To Me

Prove that the following languages are undecidable (below, M, M_1, M_2 refer to TMs).

(a) $\mathbf{REGULAR} = \{\langle M \rangle : L(M) \text{ is regular}\}$.

(b) $\mathbf{TOTAL} = \{\langle M \rangle : M \text{ halts on all inputs}\}$.

(c) $\mathbf{DOLORES} = \{\langle M_1, M_2 \rangle : \exists w \in \Sigma^* \text{ such that both } M_1(w) \text{ and } M_2(w) \text{ accept}\}$.

(Extra) Lose All Scripted Responses. Improvisation Only

Let $\mathbf{FINITE} = \{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is finite}\}$.

Show that $\mathbf{TOTAL} \leq \mathbf{FINITE}$.