15-251: Great Theoretical Ideas In Computer Science

Recitation 1

As you prepare to do the writeups next week, remember the 10 styles of proof that you should avoid (adapted from Luis Von Ahn's lecture notes):

- Proof by Stating Every Theorem in the Relevant Subject Area and Hoping for Partial Credit
- Proof by Example ("Here is a proof for $n = 2$. The general case is basically the same idea.")
- Proof by Obscurity (using enough cumbersome and complex notation that your proof is impossible to decipher)
- Proof by Lengthiness (especially powerful combined with Proof by Obscurity and Proof by Stating Every Theorem)
- Proof by Switcheroo ($p \implies q$ is true, and q is true, so p must be true, right?)
- Proof by "It is clear that..." ("Clearly, this is the worst case for our algorithm.")
- Proof by Generalization (This specific algorithm doesn't work to solve this problem, and it seems optimal, therefore no algorithm can solve it! - a particularly common and pernicious flawed proof)
- Proof by Missing Cases (Your induction hypothesis doesn't work for all cases, or your base cases are incomplete)
- Proof "by definition" ("By definition, our algorithm is $O(n)$")
- Proof by OMGWTFBBQ (enough said.)

Balanced Parentheses

In lecture, we introduced the following deductive system, where the objects are nonempty strings from $\{0, 1\}^*$.
There is one initial object, $()$.
There are two deduction rules:

- From S, deduce (S). (WRAP)
- From S and T, deduce ST. (CONCAT)

We claimed that these rules create exactly "the set of balanced strings of parentheses".
But what is that anyway?

(a) How do you reasonably define a "balanced string of parentheses"?

(b) Using this definition, prove that you can deduce a string if and only if it satisfies the definition.

Normal Induction

Prove that $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for all natural numbers n.

Wait why doesn't my induction work?

Prove that $1 + 1/4 + \cdots + 1/n^2 < 2$ for all positive natural numbers n by induction.

Be Strong with Induction

A game is played with a stack of n coins. At each step, take any stack of at least two coins, split it into two nonempty stacks, and put the two stacks back. Your score for this move is the product of the sizes. Continue play until all stacks have height 1. What is the maximum achieveable score? Is it possible to achieve two different scores?