15-251: Great Theoretical Ideas In Computer Science

Recitation 2

Announcements

- Be sure to start early on homeworks! It will be a smoother experience for all involved (less crowded office hours are better for you and us!)
- Please practice writing up your solutions beforehand. It will help with:
  - Structuring your proof on the page in an organized way.
  - Writing neatly (minimizing erasing/crossing out with practice).
  - Ensuring your solution is correct (often bugs pop up while writing).
- If you aren't sure about a piece of feedback or why you got a particular score on a problem, please ask us! The TAs initialed which problems they graded, so go to the correct TA if you are at all unsure. We're here to help!
- If you have any questions about the course overall, not restricted to homework, please ask! Again, we want to help you however we can!
- Reminder: Make sure that you're familiar with the collaboration policies on the website. The homework writing sessions and open collaboration problems are very different from many classes, so just make sure you're clear on the specifics.

A Natural System

Consider First Order Logic together with the following vocabulary:
One constant name: Z
One one-argument function name: $succ(\cdot)$
No relation names.

Pair this system with the following two axioms:
$\forall x.\, Z \neq succ(x)$
$\forall x.\, \forall y.\, succ(x) = succ(y) \implies x = y$

(a) Give an interpretation I which makes both axioms true.

(b) Define $succ^n(x)$ to be the $succ$ function applied n times to x. Is it true that in all interpretations I, for all objects x in the interpretation, $x = succ^n(Z)$ for some $n \in \mathbb{N}$?

Regular Modulus

(a) Prove that the language of binary strings whose numeric interpretation is equal to 0 $\pmod 3$ is regular. We read from the most significant bit to the least.

(b) What if we read from the least significant bit to the most?

I need more states ☹

Prove that $L_n = \{a^{nk} \mid k > 0\}$ cannot be decided by a DFA with fewer than $n + 1$ states.

Irregular languages are cooler languages

Prove that the language $L = \{ww \mid w \in \{a, b\}^*\}$ is irregular.

Closure of Regularity

(a) Prove that if L is regular, then L^C is regular.

(b) Prove that if L_1 and L_2 are regular, then $L_1 \cap L_2$ is regular.

(c) Use closure properties to show that $L = \{w \mid w \in \{0, 1\}^*$ and w has the same number of 0s and 1s$\}$ is irregular.