15-251: Great Theoretical Ideas In Computer Science

Recitation 3

Renouncements

- Review sessions will be held on Thursdays (yesterday) from 6:30-7:30 in GHC 5222.
- Attendance at recitations will be taken into account for computing your participation grade (2% total).

Calculator$^{\text{TM}}$

We say that the output of a TM is the string left on the tape if the TM halts. The input is, of course, the string on the tape before the computation starts. Can you use a TM to

- Add one to a binary number?
- Double a unary number?

Lost, Gone, and Empty

Define **EMPTY** $= \{\langle M \rangle \mid$ the set of strings accepted by M is $\emptyset\}$.

Show that **EMPTY** is undecidable.

Regular or Not? You Decide

Define **REGULAR** $= \{\langle M \rangle \mid$ the set of strings accepted by M is a regular language$\}$.

Show that **REGULAR** is undecidable.

The Post Correspondence Problem

The Post Correspondence Problem is defined as follows.

Given a set of tiles with two rows e.g.

$$\left\{ \begin{pmatrix} a & \\ a & b \end{pmatrix}, \begin{pmatrix} c & a \\ a & b \end{pmatrix}, \begin{pmatrix} b \\ c \end{pmatrix}, \begin{pmatrix} b & d \\ d & \end{pmatrix} \right\}$$

Decide: Is there a sequence of tiles (with repetition allowed) such that the string formed along the top row is the same as the string along the bottom row?

For example,

$$\begin{pmatrix} a & \\ a & b \end{pmatrix} \begin{pmatrix} b & d \\ d & \end{pmatrix} \text{ or } \begin{pmatrix} a & \\ a & b \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} \begin{pmatrix} c & a \\ a & b \end{pmatrix} \begin{pmatrix} b & d \\ d & \end{pmatrix}$$

Theorem. PCP is undecidable.

We present an outline of the proof so that

- you see an example of an undecidable problem that doesn't have to do with Turing machines, and

- you learn about configurations.

Major proof steps:

(a) Problem we want to reduce to: **ACCEPT**. Given M, w, does M accept when run on w?

(b) A configuration is a string that encodes the current computation: $w_1 q w_2$ where w_1 is the input to the left of the head, q is the state, w_2 is the input below and to the right of the head.

(c) A computation history of TM M run on w is a sequence of configurations $C_1, \ldots, C_k, \ldots$ such that C_1 is the start configuration with w, and C_{i+1} is the result of the transition function on C_i.

(d) Claim: M accepts w iff there is a computation history where the last configuration is accepting.

(e) Idea: we want a PCP representation of a computation history to look like $\binom{\#}{w}\binom{C_0}{C_1}\binom{C_1}{C_2}\binom{C_2}{C_3}\binom{C_3}{C_4}\cdots$.

(f) Changing configurations (validly) changes only a few things: state, contents of a single tape cell, head position by ± 1.

(g) Make tiles to encode these changes.