Renouncements

• Review sessions will be held on Thursdays (yesterday) from 6:30-7:30 in GHC 5222.

• Attendance at recitations will be taken into account for computing your participation grade (2% total).

Calculator

We say that the output of a TM is the string left on the tape if the TM halts. The input is, of course, the string on the tape before the computation starts. Can you use a TM to

• Add one to a binary number?

• Double a unary number?

Lost, Gone, and Empty

Define $\text{EMPTY} = \{ \langle M \rangle \mid \text{the set of strings accepted by } M \text{ is } \emptyset \}$. Show that $\text{EMPTY}$ is undecidable.

Regular or Not? You Decide

Define $\text{REGULAR} = \{ \langle M \rangle \mid \text{the set of strings accepted by } M \text{ is a regular language} \}$. Show that $\text{REGULAR}$ is undecidable.

The Post Correspondence Problem

The Post Correspondence Problem is defined as follows.

Given a set of tiles with two rows e.g.

$$\left\{ \begin{array}{ccc} (a & b) & (c & a) & (b) & (b & d) \\ (a & b) & (a & b) & (c) & (b & d) \end{array} \right\}$$

Decide: Is there a sequence of tiles (with repetition allowed) such that the string formed along the top row is the same as the string along the bottom row?

For example,

$$\begin{array}{ccc} (a & b) & (b & d) \end{array} \text{ or } \begin{array}{ccc} (a & b) & (b) & (c & a) & (b & d) \end{array}$$

Theorem. PCP is undecidable.

We present an outline of the proof so that

• you see an example of an undecidable problem that doesn’t have to do with Turing machines, and
you learn about configurations.

Major proof steps:

(a) Problem we want to reduce to: **ACCEPT**. Given $M$, $w$, does $M$ accept when run on $w$?

(b) A configuration is a string that encodes the current computation: $w_1qw_2$ where $w_1$ is the input to the left of the head, $q$ is the state, $w_2$ is the input below and to the right of the head.

(c) A computation history of TM $M$ run on $w$ is a sequence of configurations $C_1, \ldots, C_k, \ldots$ such that $C_1$ is the start configuration with $w$, and $C_{i+1}$ is the result of the transition function on $C_i$.

(d) Claim: $M$ accepts $w$ iff there is a computation history where the last configuration is accepting.

(e) Idea: we want a PCP representation of a computation history to look like $(\# w) (C_0) (C_1) (C_2) (C_3) (C_4) \cdots$.

(f) Changing configurations (validly) changes only a few things: state, contents of a single tape cell, head position by $\pm 1$.

(g) Make tiles to encode these changes.