15-251: Great Theoretical Ideas In Computer Science

Recitation 4

Announcements

- We have a midterm next week! Be sure to study!

$\mathcal{O}$, I Think I Understand Asymptotics Now

Let f, g, h be functions from $\mathbb{N}$ to $\mathbb{N}$. Prove or disprove the following:

(a) If $f \in \mathcal{O}(g)$ and $g \in \mathcal{O}(h)$, then $f \in \mathcal{O}(h)$

(b) If $f \in \mathcal{O}(g)$, then $g \in \mathcal{O}(f)$

(c) $f \in \mathcal{O}(g)$ or $f \in \Omega(g)$

Logapolyrithmic

Prove that for all $k \in \mathbb{R}^+$, $\log(n) \in \mathcal{O}(n^k)$.

Square Root: Polynomial or Exponential?

Give a polynomial time algorithm for computing $\lfloor \sqrt{n} \rfloor$, for $n \in \mathbb{N}$.

Your Guesses are Two High!

Suppose I am thinking of a number between 1 and n, and will tell you if your guess is too high, too low, or correct. However, I only allow you to guess too high once, or you lose. How quickly can you guess my number? One possible solution is to just guess incrementally from 1 to my number, which takes $\mathcal{O}(n)$ time. Can you do better?

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Consider the computational model where every operation is free except for comparisons, which have unit cost. Prove that any comparison-based sorting algorithm must cost $\Omega(n \log n)$.