15-251: Great Theoretical Ideas In Computer Science

Recitation 5

P

Let $L \subset \{0,1\}^*$. Prove that if $L \in \mathbf{P}$ then $L^* \in \mathbf{P}$.

Infinity

Let T be a TM with $|\Gamma| = g$, $|Q| = q$, and suppose it has space complexity $S(n)$.

(a) Show that if T runs for more than $qnS(n)g^{S(n)}$ steps on an input of length n, it is stuck in an infinite loop. (Recall: we are considering 2-tape TMs)

(b) Suppose that $S(n) \geq \log(n)$ and T accepts exactly L. Then, there exists a decider for L with space complexity $\mathcal{O}(S(n))$

Clique

Recall that a complete graph is a graph such that every pair of vertices has an edge. We call this a clique. Define $CLIQUE$ to be the decision problem that, given inputs G and k, determines whether G contains a clique of k vertices as a subgraph. Show that $CLIQUE \in$ **PSPACE**

Path

Define $PATH$ to be the decision problem that, given inputs G, s, t, determines whether there exists a path from s to t in graph G. Find a decision algorithm for $PATH$ with space complexity $\mathcal{O}(\log^2(n))$

Irregular Space

Recall that a regular language is a language which a DFA can decide. It is also true that a language is regular if and only if a Turing Machine with less than $\Omega(\log(\log(n)))$ space complexity can decide it. This theorem is beyond the scope of this class, but motivates the following problem: Is there an irregular language which can be decided with $\mathcal{O}(\log(\log(n)))$ space?

(a) Let our alphabet be $\{0, 1, \#\}$ and define $bin(n)$ to be the binary representation of n.
Prove that $L = \{bin(1)\#bin(2)\# \cdots \#bin(n) \mid n \in \mathbb{N}\}$ is irregular.

(b) Prove that L can be decided by a TM with space complexity $\mathcal{O}(\log(\log(n)))$.