15-251: Great Theoretical Ideas In Computer Science

Recitation 6

Draw

Let $\mathcal{T}_n$ denote the set of trees on the vertex set $[n] = \{1, 2, ..., n\}$.

a) Draw all the distinct elements of $\mathcal{T}_3$.

b) Compute $|\mathcal{T}_4|$.

Degrees and paths

Suppose that a graph G has minimal degree d. Prove that G has a path of length d.

Marriage

Consider the following decision problem and come up with a polynomial time algorithm.

Input : An instance of Stable Matching problem

Output : True if there exists a unique stable matching. False otherwise.

Self-Complements

Complement of a graph $G = (V, E)$ is the graph $\overline{G} = (V, \overline{E})$ where $\overline{E}$ is made up of all vertex-pairs that are not in E.

A graph $G_0 = (V_0, E_0)$ is isomorphic to $G_1 = (V_1, E_1)$ if there exists a bijection $f : V_0 \to V_1$ such that $\{v_1, u_1\} \in E_1$ if and only if $\{v_1, u_1\} = \{f(v_0), f(u_0)\}$ for some $\{v_0, u_o\} \in E_0$.

A graph G is said to be self-complementary if it is isomorphic to its complement. Prove that there exists a self complementary graph $G = (V, E)$ on n vertices if and only if $n \equiv 0$ or $1 \bmod 4$.