15-251: Great Theoretical Ideas In Computer Science

Recitation 9

Warmup

Let p be a prime. Then for $1 \leq m \leq p-1$, $\binom{p}{m} \equiv 0 \bmod p$

More floor functions!

Let $p \geq 5$ be a prime and let $k = \lfloor \frac{2p}{3} \rfloor$. Show that

$$\sum_{m=1}^{k} \binom{p}{m} \equiv 0 \bmod p^2$$

Matrix Multiplication

Given three n by n matrices A, B, C, write a randomized algorithm which checks if $AB = C$. Specifically, if $AB = C$, your algorithm must return true. If $AB \neq C$, your algorithm must return false with probability at least 1/2. Your algorithm should run in $O(n^2)$ time.

Quickselect

In lecture, we saw an implementation of randomized Quicksort. Here, we will analyze a slightly different problem: how do you select the kth smallest element of a list? The algorithm we use is very similar to Quicksort. In the same way as before, we pick a pivot and partition the list into less than and greater than. The point where this algorithm differs is in the recursive step. For Quicksort, we had to recurse on both sublists. For Quickselect, we recurse into the one which contains the kth element (by observing the size of the two lists and picking the right one).

Let $a_1, \cdots, a_n$ be the input list and $s_1, \cdots, s_n$ be the sorted list. Suppose we are looking for s_k. We want to analyze the probability that s_i and s_j are compared, for $1 \leq i < j \leq n$. Be sure to split up into cases based on their relation to k.

Aside: It turns out that when we have these probabilities, we can prove that Quickselect is expected $O(n)$ time. We will not prove this in recitation.