CMU 15-252 **Spring 2017**

HOMEWORK 1
DUE FEBRUARY 3RD IN CLASS

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1. In this question we slightly modify the pancake sorting problem so that the allowed operation is to take any contiguous set of pancakes and flip them in place (they need not be on the top of the stack).

   Recall from the lecture that a pair of adjacent pancakes is *bad* if their values differ by more than 1, as such a pair needs to be broken in the end. Put an imaginary 0 sentinel at the top and an imaginary $n+1$ sentinel at the bottom of the stack (you cannot move them around), so the first pancake contributes a *bad pair* if its value is not 1, and the last pancake also contributes a bad pair if its value is not n. Imagine we put a *separator* between the pancakes in every bad pair.

   The sequence of pancakes between any two consecutive separators is called a *patch*. A patch is called *decreasing* if the values decrease from top to bottom, and called *increasing* otherwise. For example, in the stack (4 3 2 5 6 1), the patches are (4 3 2), (5 6) and (1). The patch (4 3 2) is decreasing, and (5 6) is increasing. Singleton patches are considered to be decreasing, except for the first and the last pancake. So in the example, (1) is considered increasing.

   (a) Show that every stack of pancakes that has a decreasing patch also has a flip that reduces the number of bad pairs by at least 1.

   (b) Use part (a) to show the following: For any stack of n pancakes with $b \geq 2$ bad pairs, if t is the number of flips required to sort it, then $b/2 \leq t \leq 2b - 3$.