1. (a) Let $L \subseteq \Sigma^*$ be a regular language. Is

$$L' = \{ u \in \Sigma^* : \exists v \in \Sigma^* \text{ such that } uv \in L \text{ and } |u| = |v| \}$$

necessarily regular? Prove your answer.

(b) Let $L \subseteq \Sigma^*$ be a regular language. Is

$$L'' = \{ uv \in \Sigma^* : \exists w \in \Sigma^* \text{ such that } uwv \in L \text{ and } |u| = |v| = |w| \}$$

necessarily regular? Prove your answer.

As a hint, we give the yes/no answers to these questions on the next page. Of course, you still have to provide proofs for those answers.
(a) Yes. (b) No.