

HOMEWORK 5
DUE MARCH 3RD IN CLASS

1. Show that if a TM decides a language using $S(n)$ space (where $S(n) \geq \log n$), then it decides the language in $2^{O(S(n))}$ time.
2. Let $\Sigma = \{0, 1, \#\}$. Consider the language $L \subseteq \Sigma^*$ consisting of the words

0#1#
00#01#10#11#
000#001#010#011#100#101#110#111#
⋮

(If the pattern is not clear, please don't hesitate to ask.) Show that (i) this language can be decided using $O(\log \log n)$ space; (ii) the language is not regular.

Fun fact (which you do not need to prove): If a language can be decided using $o(\log \log n)$ space,¹ then it must be regular.

¹Little-o notation: https://en.wikipedia.org/wiki/Big_O_notation#Little-o_notation