15-251: Great Theoretical Ideas In Computer Science

Recitation 10 : Approximation Algorithms

- Solution Sessions for HW8 **Friday 5-6pm** and **Saturday 2-3pm** in **GHC 4301**
- Conceptual Office Hours on **Friday** from **6-8pm** in **Gates 5 Carrel 1 (Double Carrel)**.

Mostly Done in Lecture

- The Cook-Levin theorem says that CIRCUIT-SAT is **NP-complete**.
- The goal of an optimization problem is to find the minimum (or maximum) value under some constraints.
- OPT(I) is the value of the optimal solution to an instance I of an optimization problem.
- We say an algorithm $\mathcal{A}$ for an optimization problem is a factor-α approximation if for all instances I of the problem $\mathcal{A}$ outputs a solution that is at least as good as $\alpha \cdot \text{OPT}(I)$.

Partly Split

Here is an optimization version of the NP-hard PARTITION problem. Given a set X of positive integers, separate X into disjoint subsets A, B. Minimize $\max(\sum A, \sum B)$.

Determine the approximation ratio of the following greedy algorithm: while you have numbers remaining, put one into the smaller-sized subset.

Cutting it Close

A cut on a graph G is an assignment of vertices into two sets $A, B \subseteq V$ (or assignment of colors to the vertices). The value of a cut is the number of edges e such that one endpoint is in A and one endpoint is in B. MAX-CUT is the language of $\langle G, k \rangle$ such that there exists a cut of size at least k in G.

MAX-CUT is NP-complete (this is a hard reduction). However, it can also be expressed as an optimization problem: maximize k such that G has a cut of size k.

Find a factor-$\frac{1}{2}$ approximation to MAX-CUT.