15-251: Great Theoretical Ideas In Computer Science

Recitation 4

Announcements

Be sure to take advantage of the following resources :

- Homework Solution Sessions - Friday 5p-6p, Saturday 2p-3p in GHC 4301
- Conceptual office hours (no HW help) - Friday 6p-8p in Gates 5 Carrel 1
- If you are struggling with course material, please come to small groups/conceptual office hours or set up a meeting with your TA as soon as possible.

These Decidable Definitions Have Undecidable Ends

- **Church-Turing Thesis:** Any natural/reasonable notion of computation can be simulated by a TM.
- A **decider** is a TM that halts on all inputs.
- A language L is **undecidable** if there is no TM M that halts on all inputs such that $M(x)$ accepts if and only if $x \in L$.
- $A \leq B$: It is possible to decide A using an algorithm that decides B as a subroutine.

Freeze All Automata Functions

Prove that the following languages are decidable by reducing it to $\mathbf{EMPTY_{DFA}}$.

(a) $\mathbf{NO-ODD-ONES} = \{\langle D\rangle : D$ does not accept any string containing an odd number of 1's$\}$

(b) $\mathbf{INF_{DFA}} = \{\langle D\rangle : D$ is a DFA with $L(D)$ infinite$\}$.

Hint: Consider a DFA with k states that accepts some string with more than k characters.

Doesn't Look Like Anything (Decidable) To Me

Prove that the following languages are undecidable (below, M, M_1, M_2 refer to TMs).

(a) $\mathbf{REGULAR} = \{\langle M\rangle : L(M)$ is regular$\}$.

(b) $\mathbf{TOTAL} = \{\langle M\rangle : M$ halts on all inputs$\}$.

(c) $\mathbf{DOLORES} = \{\langle M_1, M_2\rangle : \exists w \in \Sigma^*$ such that both $M_1(w)$ and $M_2(w)$ accept$\}$.

(Extra) Lose All Scripted Responses. Improvisation Only

Let $\mathbf{FINITE} = \{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is finite}\}$.

Show that $\mathbf{TOTAL} \leq \mathbf{FINITE}$.

(Bonus) The Maize is not Meant For You

Josh Corn is trying to write a program P such that given a natural number n, $P(n)$ is the most number of steps a TM on n states can take before halting. Show that this is not possible.