15-251: Great Theoretical Ideas In Computer Science

Recitation 9 : P and NP

News Post

- NP and NP-completeness Review Session **Saturday, March 25** from **11am-12:30pm** in **GHC 4301**
- Solution Sessions for both HW6 and HW7 **Friday 5-7pm** in **GHC 4102** and **Saturday 1-3pm** in **GHC 4301**
- Conceptual Office Hours on **Friday** from **7-8pm** in **Gates 5 Carrel 1 (Double Carrel)**.
- The deadline for resubmissions for homeworks 6 and 7 has been changed to **10pm on Thursday, March 30**.

New Phrases

- We say a language is in **P** if there exists a polynomial time algorithm that decides the language
- We say a problem is in **NP** if there exists a polynomial time verifier TM V such that for all $x \in \Sigma^*$, x is in L if and only if there exists a polynomial length certificate u such that $V(x, u) = 1$.
- We say there is a **polynomial-time many-one reduction** from A to B if there is a **polynomial-time** computable function $f : \Sigma^* \to \Sigma^*$ such that $x \in A$ if and only if $f(x) \in B$. We write this as $A \leq_m^P B$. (We also refer to these reductions as Karp reductions.)
- A problem Y is **NP-hard** if for every problem $X \in$ **NP**, $X \leq_m^P Y$.
- A problem is **NP-complete** if it is both in **NP** and **NP-hard**.

NP is Not Not Polynomial

Show that **P** is contained in **NP**.

No Privacy

DOUBLE-CLIQUE: Given a graph $G = (V, E)$ and a natural number k, does G contain two vertex-disjoint cliques of size k each?

Show DOUBLE-CLIQUE is **NP-Complete**.

No Pun

POP-SET: Given a graph $G = (V, E)$, and a natural number k, does there exist a subset $U \subseteq V$ with $|U| \leq k$ such that every edge $e \in E$ has at least one of its endpoints in U? (Note: On Homework 7 we referred to such a U as a popular set.)

Show POP-SET is **NP-hard**. (Hint: Reduce from 3SAT)