

# 15-251

## Great Theoretical Ideas in Computer Science

### Lecture 1.5: On proofs + How to succeed in 251

*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E}[f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\hat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\hat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

□



Jan 18th, 2017

*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E} [f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\widehat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\widehat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

□

1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E}[f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\hat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\hat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

□

1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

# Is this a legit proof?

## Proposition:

Start with any number.

If the number is even, divide it by 2.

If it is odd, multiply it by 3 and add 1.

If you repeat this process, it will lead you to 4, 2, 1.

## Proof:

Many people have tried this, and no one came up with a counter-example.



# Is this a legit proof?

## ~~Proposition~~ Collatz Conjecture:

Start with any number.

If the number is even, divide it by 2.

If it is odd, multiply it by 3 and add 1.

If you repeat this process, it will lead you to 4, 2, 1.

### Proof:

Many people have tried this, and no one came up with a counter-example.



# Is this a legit proof?

## Proposition:

$313(x^3 + y^3) = z^3$  has no solution for  $x, y, z \in \mathbb{Z}^+$ .

## Proof:

Using a computer, we were able to verify that there is no solution for numbers with  $< 500$  digits.



# Is this a legit proof?

~~Proposition:~~

$313(x^3 + y^3) = z^3$  has no solution for  $x, y, z \in \mathbb{Z}^+$ .

~~Proof:~~

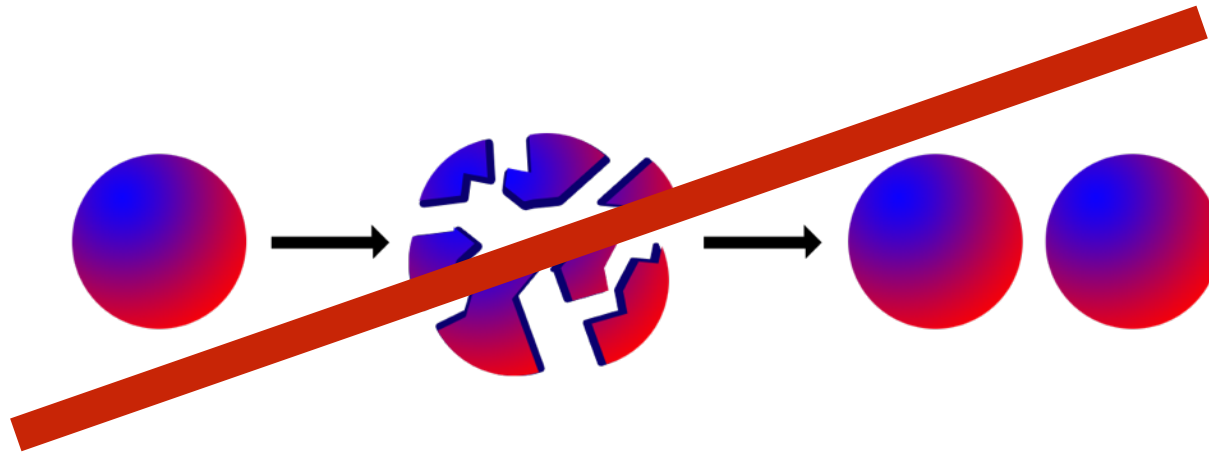
~~Using a computer, we were able to verify that there is no solution for numbers with  $< 500$  digits.~~



# Is this a legit proof?

## Proposition:

Given a solid ball in 3 dimensional space, there is no way to decompose it into a finite number of disjoint subsets, which can be put together to form two identical copies of the original ball.



## Proof:

Obvious.

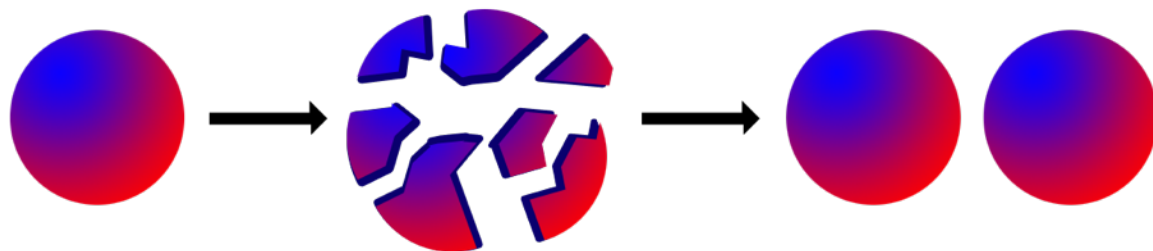




# Is this a legit proof?

## Banach-Tarski Theorem:

Given a solid ball in 3 dimensional space, there **is a** way to decompose it into a finite number of disjoint subsets, which can be put together to form two identical copies of the original ball.



## Proof:

Uses group theory... The pieces are such weird scatterings of points that they have no meaningful “volume”...

# Is this a legit proof?

**Proposition:**

$$1 + 1 = 2$$

**Proof:**

This is obvious?

# Is this a legit proof?

**Proposition:**

$$1 + 1 = 2$$

**Proof:**

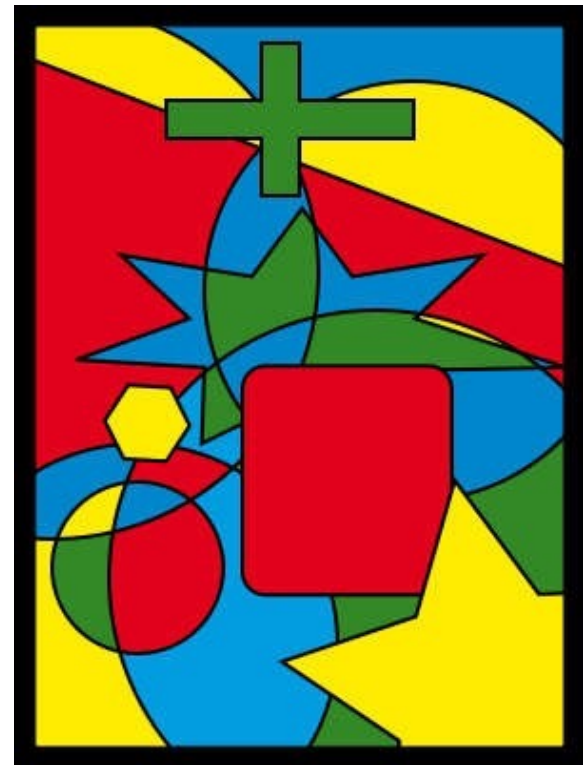
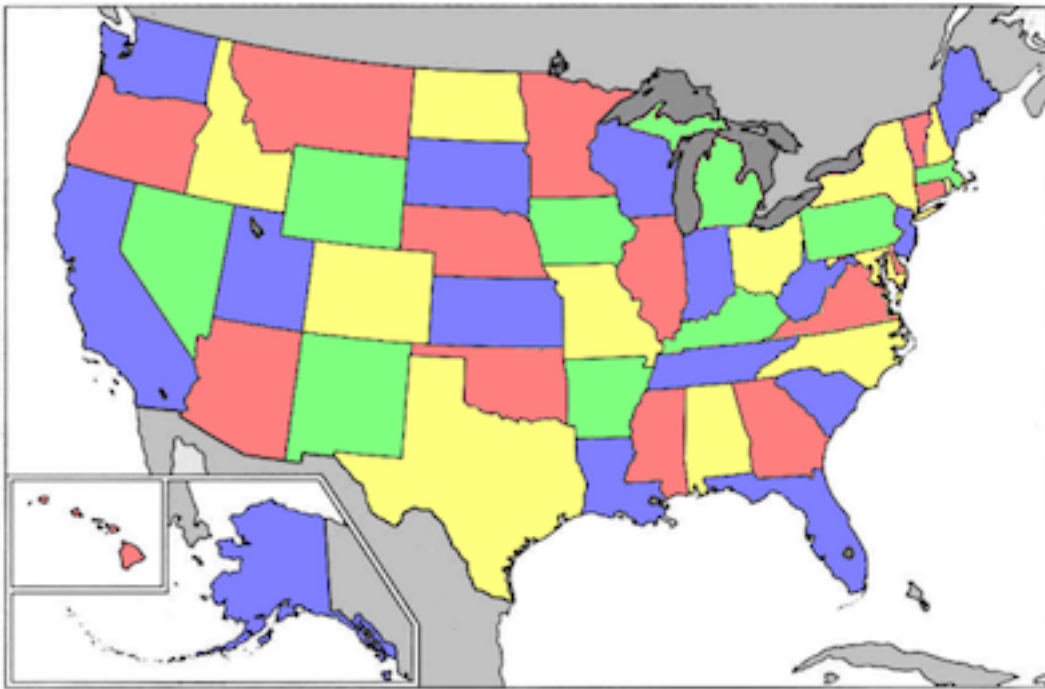
This is obvious!



# The story of 4 color theorem

## 1852 Conjecture:

Any 2-d map of regions can be colored with 4 colors so that no adjacent regions get the same color.



# The story of 4 color theorem

**1879:** Proved by **Kempe** in *American Journal of Mathematics*  
(was widely acclaimed)

**1880:** Alternate proof by **Tait** in *Trans. Roy. Soc. Edinburgh*

**1890:** **Heawood** finds a bug in **Kempe's** proof

**1891:** **Petersen** finds a bug in **Tait's** proof

**1969:** **Heesch** showed the theorem could in principle be reduced to checking a large number of cases.

**1976:** **Appel** and **Haken** wrote a massive amount of code to compute and then check 1936 cases.  
(1200 hours of computer time)



# The story of 4 color theorem

Much controversy at the time. Is this a proof?

What do you think?

Arguments against:

- no human could ever hand-check the cases
- maybe there is a bug in the code
- maybe there is a bug in the compiler
- maybe there is a bug in the hardware
- no “insight” is derived

**1997:** Simpler computer proof by  
Robertson, Sanders, Seymour, Thomas

# What is a mathematical proof?

*inference rules like* 
$$\frac{P, P \implies Q}{Q}$$

A mathematical **proof of a proposition** is a chain of **logical deductions** starting from a set of **axioms** and leading to the **proposition**.

*propositions accepted to be true*

*a statement that is true or false*

# Euclidian geometry

## 5 AXIOMS

1. Any two points can be joined by exactly one line segment.
2. Any line segment can be extended into one line.
3. Given any point  $P$  and length  $r$ , there is a circle of radius  $r$  and center  $P$ .

4. Any two right angles are congruent.

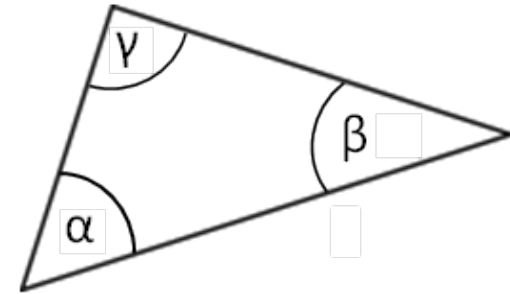
5. If a line  $L$  intersects two lines  $M$  and  $N$ , and if the interior angles on one side of  $L$  add up to less than two right angles, then  $M$  and  $N$  intersect on that side of  $L$ .



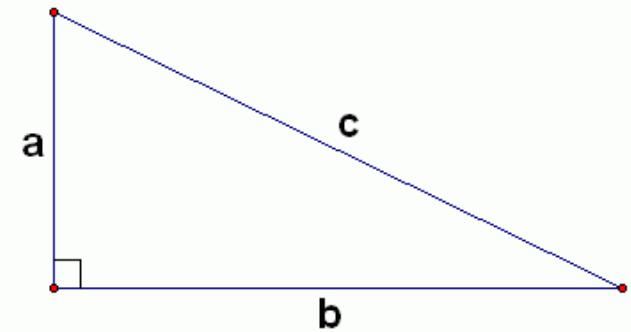


# Euclidian geometry

## Triangle Angle Sum Theorem

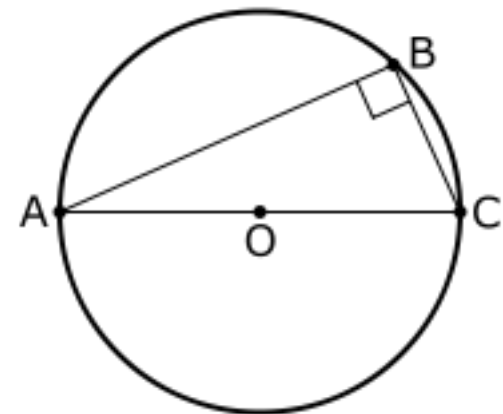


## Pythagorean Theorem



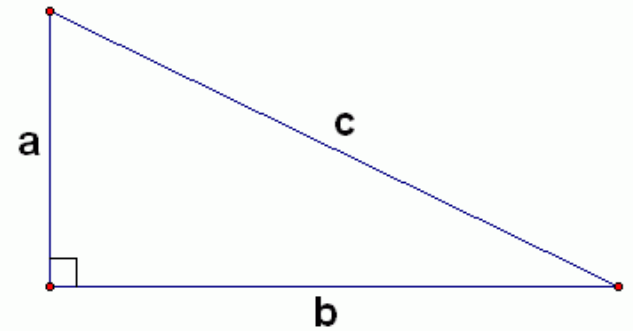
$$a^2 + b^2 = c^2$$

## Thales' Theorem



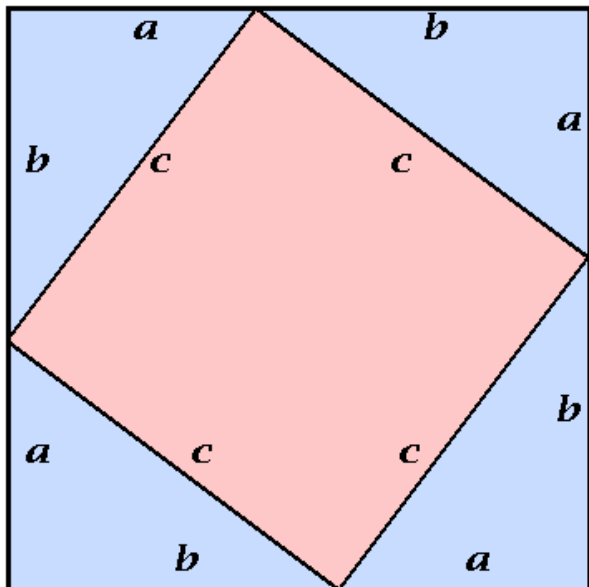
# Euclidian geometry

## Pythagorean Theorem



$$a^2 + b^2 = c^2$$

## Proof:



$$\begin{aligned}c^2 &= (a + b)^2 - 2ab \\ &= a^2 + b^2.\end{aligned}$$

Looks legit.



# Proof that square-root(2) is irrational

1. Suppose  $\sqrt{2}$  is rational.

Then we can find  $a, b \in \mathbb{N}$  such that  $\sqrt{2} = a/b$ .

2. If  $\sqrt{2} = a/b$  then  $\sqrt{2} = r/s$ ,  
where  $r$  and  $s$  are not both even.

3. If  $\sqrt{2} = r/s$  then  $2 = r^2/s^2$ .

4. If  $2 = r^2/s^2$  then  $2s^2 = r^2$ .

5. If  $2s^2 = r^2$  then  $r^2$  is even, which means  $r$  is even.

6. If  $r$  is even,  $r = 2t$  for some  $t \in \mathbb{N}$ .

7. If  $2s^2 = r^2$  and  $r = 2t$  then  $2s^2 = 4t^2$  and so  $s^2 = 2t^2$ .

8. If  $s^2 = 2t^2$  then  $s^2$  is even, and so  $s$  is even.

# Proof that square-root(2) is irrational

1. Suppose  $\sqrt{2}$  is rational.

Then we can find  $a, b \in \mathbb{N}$  such that  $\sqrt{2} = a/b$ .

2. If  $\sqrt{2} = a/b$  then  $\sqrt{2} = r/s$ ,  
where  $r$  and  $s$  are not both even.

3. If  $\sqrt{2} = r/s$  then  $2 = r^2/s^2$ .

4. If  $2 = r^2/s^2$  then  $2s^2 = r^2$ .

5. If  $2s^2 = r^2$  then  $r^2$  is even, which means  $r$  is even.

6. If  $r$  is even,  $r = 2t$  for some  $t \in \mathbb{N}$ .

7. If  $2s^2 = r^2$  and  $r = 2t$  then  $2s^2 = 4t^2$  and so  $s^2 = 2t^2$ .

8. If  $s^2 = 2t^2$  then  $s^2$  is even, and so  $s$  is even.

9. Contradiction is reached.

# Proof that square-root(2) is irrational

1. Suppose  $\sqrt{2}$  is rational.

Then we can find  $a, b \in \mathbb{N}$  such that  $\sqrt{2} = a/b$ .

2. If  $\sqrt{2} = a/b$  then  $\sqrt{2} = r/s$ ,  
where  $r$  and  $s$  are not both even.

3. If  $\sqrt{2} = r/s$  then  $2 = r^2/s^2$ .

4. If  $2 = r^2/s^2$  then  $2s^2 = r^2$ .

5. If  $2s^2 = r^2$  then  $r^2$  is even, which means  $r$  is even.

6. If  $r$  is even,  $r = 2t$  for some  $t \in \mathbb{N}$ .

7. If  $2s^2 = r^2$  and  $r = 2t$  then  $2s^2 = 4t^2$  and so  $s^2 = 2t^2$ .

8. If  $s^2 = 2t^2$  then  $s^2$  is even, and so  $s$  is even.

9. Contradiction is reached.

# Proof that square-root(2) is irrational

5a.  $r^2$  is even. Suppose  $r$  is odd.

5b. So there is a number  $t$  such that  $r = 2t + 1$ .

5c. So  $r^2 = (2t + 1)^2 = 4t^2 + 4t + 1$ .

5d.  $4t^2 + 4t + 1 = 2(2t^2 + 2t) + 1$ , which is odd.

5e. So  $r^2$  is odd.

5f. Contradiction is reached.

Odd number means not a multiple of 2.

Is every number a multiple of 2 or one more than a multiple of 2?

# Proof that square-root(2) is irrational

Odd number means not a multiple of 2.

Is every number a multiple of 2 or one more than a multiple of 2?

5b1. Call a number  $r$  **good** if  $r = 2t$  or  $r = 2t + 1$  for some  $t$ .

If  $r = 2t$ ,  $r + 1 = 2t + 1$ .

If  $r = 2t + 1$ ,  $r + 1 = 2t + 2 = 2(t + 1)$ .

Either way,  $r + 1$  is also **good**.

5b2. 1 is **good** since  $1 = 0 + 1 = (0 \cdot 2) + 1$ .

5b3. Applying 5b1 repeatedly, 2, 3, 4, ... are all **good**.

# Proof that square-root(2) is irrational

## Axiom of induction:

Suppose for every positive integer  $n$ , there is a statement  $S(n)$ .

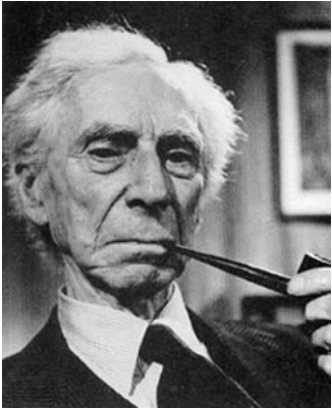
If  $S(1)$  is true, and  $S(n) \implies S(n + 1)$  for any  $n$ ,  
then  $S(n)$  is true for every  $n$ .



Can every mathematical theorem be derived from a set of agreed upon axioms?

# Formalizing math proofs

## Principia Mathematica Volume 2



Russell



Whitehead

86 CARDINAL ARITHMETIC [PART III]

\*110·632.  $\vdash : \mu \in NC . \supset . \mu +_c 1 = \hat{\xi} \{ (\exists y) . y \in \xi . \xi - t'y \in sm''\mu \}$   
*Dem.*  
 $\vdash . *110·631 . *51·211·22 . \supset$   
 $\vdash : Hp . \supset . \mu +_c 1 = \hat{\xi} \{ (\exists \gamma, y) . \gamma \in sm''\mu . y \in \xi . \gamma = \xi - t'y \}$   
[\*13·195]  $= \hat{\xi} \{ (\exists y) . y \in \xi . \xi - t'y \in sm''\mu \} : \supset \vdash . Prop$

\*110·64.  $\vdash . 0 +_c 0 = 0$  [\*110·62]  
\*110·641.  $\vdash . 1 +_c 0 = 0 +_c 1 = 1$  [\*110·51·61 . \*101·2]  
\*110·642.  $\vdash . 2 +_c 0 = 0 +_c 2 = 2$  [\*110·51·61 . \*101·31]

**\*110·643.  $\vdash . 1 +_c 1 = 2$**

*Dem.*  
 $\vdash . *110·632 . *101·21·28 . \supset$   
 $\vdash . 1 +_c 1 = \hat{\xi} \{ (\exists y) . y \in \xi . \xi - t'y \in 1 \}$   
[\*54·3]  $= 2 . \supset \vdash . Prop$

**The above proposition is occasionally useful.** It is used at least three times, in \*113·66 and \*120·123·472.

Writing a proof like this  
is like writing a computer program in machine language.

# LOGICOMIX



AN EPIC SEARCH FOR TRUTH

APOSTOLOS DOXIADIS AND CHRISTOS H. PAPADIMITRIOU

ART BY ALECOS PAPADATOS AND ANNIE DI DONNA

# Formalizing math proofs

It became generally agreed that you **could** rigorously formalize mathematical proofs.

But nobody wants to.  
(by hand, at least)

**Interesting consequence:**

**Proofs can be verified mechanically.**

**One last story**



Lord Wacker von Wackenfels  
(1550 - 1619)

# Kepler Conjecture

1611: Kepler as a New Year's present (!) for his patron, Lord Wacker von Wackenfels, wrote a paper with the following conjecture.



The densest way to pack oranges is like this:



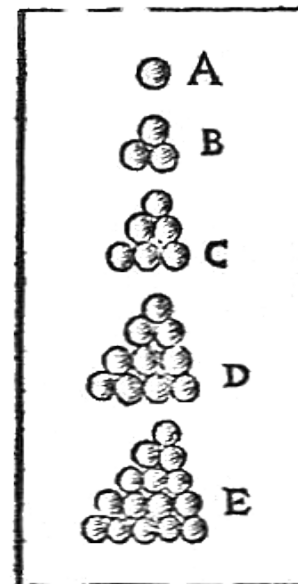
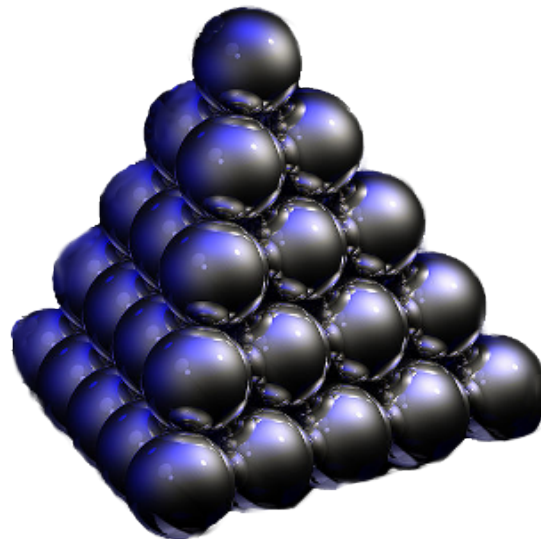


# Kepler Conjecture

1611: Kepler as a New Year's present (!) for his patron, Lord Wacker von Wackenfels, wrote a paper with the following conjecture.



The densest way to pack spheres is like this:



# Kepler Conjecture

**2005:** Pittsburgher **Tom Hales** submits a 120 page proof in *Annals of Mathematics*.

Plus code to solve 100,000 distinct optimization problems, taking 2000 hours computer time.



*Annals* recruited a team of 20 refs.  
They worked for 4 years.  
Some quit. Some retired. One died.  
In the end, they gave up.

They said they were “99% sure” it was a proof.

# Kepler Conjecture



Hales: “I will code up a completely formal axiomatic deductive proof, checkable by a computer.”

**2004 - 2014:** Open source “Project Flyspeck”:

**2015:** Hales and 21 collaborators publish  
“A formal proof of the Kepler conjecture”.

# Formally proved theorems

Fundamental Theorem of Calculus (*Harrison*)

Fundamental Theorem of Algebra (*Milewski*)

Prime Number Theorem (*Avigad @ CMU, et al.*)

Gödel's Incompleteness Theorem (*Shankar*)

Jordan Curve Theorem (*Hales*)

Brouwer Fixed Point Theorem (*Harrison*)

Four Color Theorem (*Gonthier*)

Feit-Thompson Theorem (*Gonthier*)

Kepler Conjecture (*Hales++*)

## Summary / Bottom Line

In math, there are agreed upon rigorous rules for deduction. Proofs are either right or wrong.

Nevertheless, what constitutes an acceptable proof is a social construction.

(But computer science can help.)

## What does this all mean for 15-25 I?

A proof is an argument that can withstand all criticisms from a highly caffeinated adversary (your TA).



*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E}[f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\widehat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\widehat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

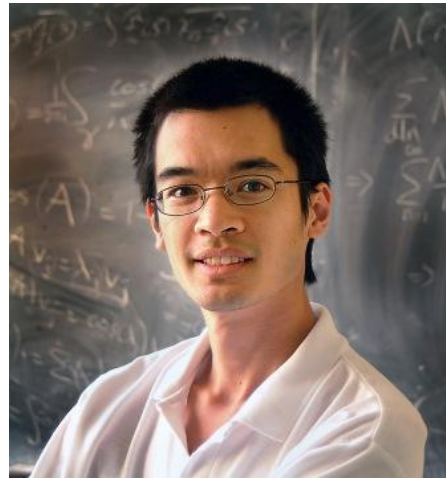
□

1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

# How do you find a proof?



No Eureka effect



Terence Tao

(Fields Medalist,  
“MacArthur Genius”,  
...)

I don't have any magical ability. ... When I was a kid, I had a romanticized notion of mathematics, that hard problems were solved in 'Eureka' moments of inspiration. [But] with me, it's always, 'Let's try this. That gets me part of the way, or that doesn't work. Now let's try this. Oh, there's a little shortcut here.' You work on it long enough and you happen to make progress towards a hard problem by a back door at some point. At the end, it's usually, 'Oh, I've solved the problem.'



# How do you find a proof?

## Some suggestions:

Make 1% progress for 100 days.

(Make 17% progress for 6 days.)

Give breaks, let the unconscious brain do some work.

Figure out some meaningful special cases (e.g.  $n=1$ ,  $n=2$ ).

Put yourself in the mind of the adversary.

(What are the worst-case examples/scenarios?)

Develop good notation.

Use paper, draw pictures.

Collaborate.

# How do you find a proof?

## Some suggestions:

Try different proof techniques.

- contrapositive  $P \implies Q \iff \neg Q \implies \neg P$
- contradiction
- induction
- case analysis

*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E} [f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\hat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\hat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

□

1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

# How do you write a proof?

- A proof is an essay, not a calculation!
- State your proof strategy.
- For long/complicated proofs, explain the proof idea first.
- Keep a linear flow.
- Introduce notation when useful. Draw diagrams/pictures.
- Structure long proofs.
- Be careful using the words “obviously” and “clearly”.  
(obvious: a proof of it springs to mind immediately.)
- Be careful using the words “it”, “that”, “this”, etc.
- Finish: tie everything together and explain why the result follows.

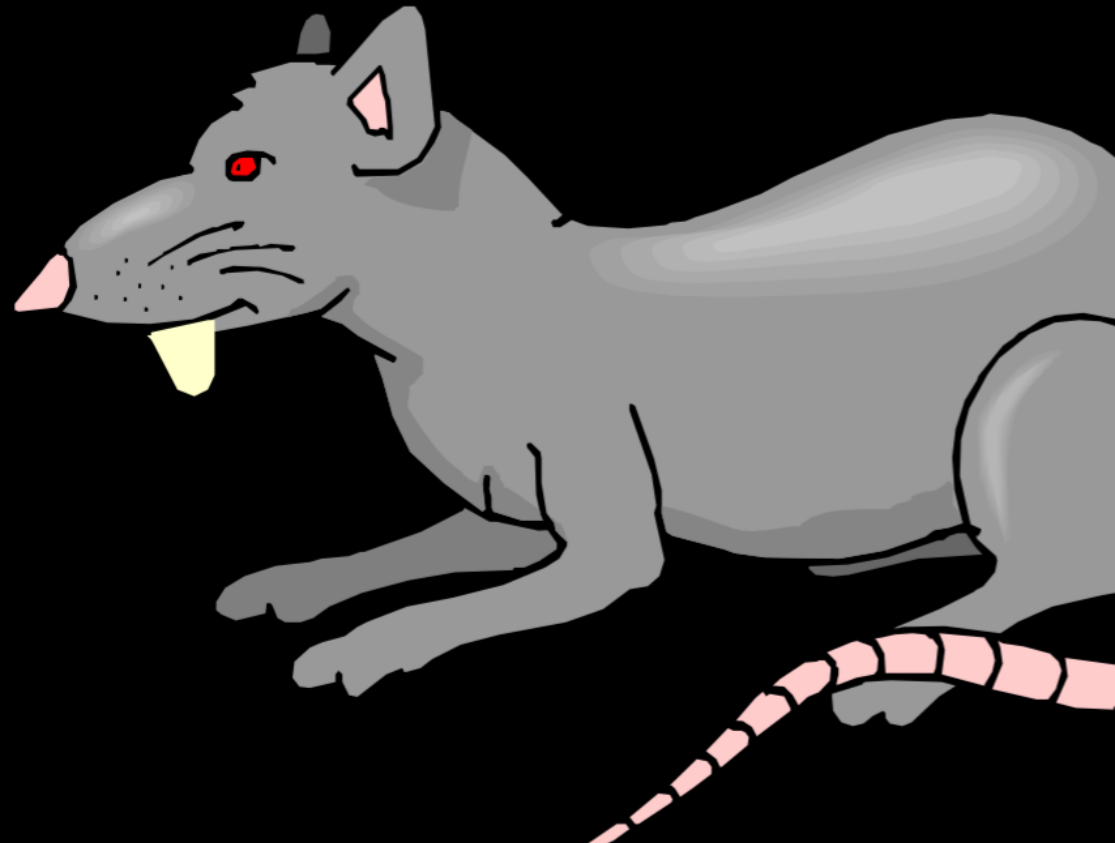
## PART 2

How to succeed in 15-251

# Understand the challenge

What did our brains evolve to do?

What were our brains “intelligently designed” to do?



# Understand the challenge

1. Math is hard!

2. The undergrad CS curriculum at CMU is challenging!

- **You** are some of the most talented (CS) students in the world.
- **We** (the faculty) hope your CMU experience pushes you to excel at a level beyond your imagination.



# Understand the challenge

## **Learning by immersion**

**You learn best by doing (and doing a lot)!**

# Embrace the challenge

## What Kind of Mindset Do You Have?



I can learn anything I want to.  
When I'm frustrated, I persevere.  
I want to challenge myself.  
When I fail, I learn.  
Tell me I try hard.  
If you succeed, I'm inspired.  
My effort and attitude determine everything.



I'm either good at it, or I'm not.  
When I'm frustrated, I give up.  
I don't like to be challenged.  
When I fail, I'm no good.  
Tell me I'm smart.  
If you succeed, I feel threatened.  
My abilities determine everything.

# **ADVICE FROM PREVIOUS 15-251 STUDENTS**

# Time management

*It's not so bad if you respect the amount of time the course deserves.*

*If you leave enough time for 25% work, it won't be stressful, it'll just be fun. But you have to leave yourself a good amount of time.*

*If you consistently put in the effort and stay on top of the assignments, you will be fine. For tests, nothing beats practice problems.*

# Time management

*Be proactive and don't procrastinate! Take advantage of office hours!*

*Always start the hw early, even if it's just reading through the problems. Start the day it's given out. It takes a lot of time to just think about the problems and ruminate over the possible paths to a solution.*

*Start the homework early and write it out!! (No matter how painful it might be).*

# Time management

*Start hw early and go to office hours often. Don't slack off because if you slack off one week you have to pick up the slack the following week and it accumulates. Take advantage of the practice problems each week; don't rely only on the homework.*

*Look at the homework early and let the problems stay in the back of your mind. Take enough time to understand what the question is exactly asking, before attempting to solve it.*

# Time management

*Start homework early. Be nice to your group. Meet people out of your group to collaborate with on open problems. Go to lecture and recitation.*

*Don't stare at one problem for too long, and don't try to meet with your group for many hours at a time or you'll all get bitchy at each other. Also go to office hours!*

# Get help

*Go to office hours. They are helpful.*

*get ur shit together and don't be afraid to ask for help.*



# Get help

*GO TO THE PROF'S OFFICE HOURS AT THE BEGINNING OF THE SEMESTER.*

*Don't give up! go to office hours & befriend the course staff!!*

*251 is hard, but is fun and doable. Ask TA for help, go to office hours. They are really helpful.*

# Review (early). Understand.

*Review the material before recitation.*

*Review the materials the night right after the lecture, don't wait until the day you start homework to do that, things are fleeting in 251.*

*Make sure to understand the topics fully before starting hw.*

# Review (early). Understand.

*Read the notes and slides until you completely understand them, then understand the questions on the homework completely before trying to come up with an answer.*

*Spend lots of time listening to lecture and getting down the concept - and don't be scared! Embrace the challenge!*

# Review (early). Understand.

*Make sure to really understand the definitions and basics.*

*Pay attention to definitions and rigorousness of proof.*

*Understand course material before starting doing homework.  
Definitions are really really important for this class*

# Review (early). Understand.

*Read lecture notes carefully before starting homework*

*Read the notes and slides until you completely understand them, then understand the questions on the homework completely before trying to come up with an answer.*

*Read the notes instead of the slides, ask questions.*

# Diligence with proofs

*Always be on the lookout for edge cases that may fail. Don't accept a solution as correct until you have slept on it and convinced yourself for more than 24 hours.*

*Double check solutions on homeworks and tests, even if you're confident they're correct.*

*Spend a lot of time practicing proofs at the beginning. Write out (at the beginning) literally every justification for every proof step.*

# Diligence with proofs

*Learn how to look at a proof and determine whether it works or if it is 'hand-wavy'. Being inquisitive helps. Many things that seem obvious are not, and vice versa. Take the time to write formal proofs for the homework problems because there are always little things that you have to think about and you may not think about unless you actually sit down and write up the solutions.*

*Make sure to write proofs very carefully, with lots of detail. Make sure you know what you're doing at every step.*

# Group

*Find a good group.*

*Find a good group, and expect to be spending a lot of time with them. A lot of the success or failure in the class will come from how well you can work together with your group so that during homework sessions you can all learn something. There will absolutely be problems or concepts which you don't understand as well as someone else in your group, and vice versa. That way you can teach each other, which is ideal. Also, if you get stumped, absolutely attend office hours. The TA's are generally quite helpful.*



# Group

*Don't just sit by and let your group do the group problems for you. Understand them going in so you can provide insight.*

*If you can, find a group of peers who not only have compatible schedules, but also have about the same experience/ability solving these types of problems. An imbalanced group can be frustrating for everyone.*

*Find a group that you're comfortable with (ideally with people who are around the same level of thinking as you), start hws early, make sure to schedule in such a way to allow for a good amount of time for this course.*

# Group

*Be wary of joining a group with a lot more experience than you on these topics, they may want to go faster than you can reasonably learn at.*

*Don't be afraid to switch groups.*

*Choose your group carefully; make sure that you feel comfortable calling your group members lazy bums if necessary.*

## Other comments

*Find a good group before the course starts. The quality of your group affects how much you get out of 25 I so much. When it comes to homework, quality over quantity. There's no point half understanding a bunch of the problems and getting terrible grades on every problem. Don't be afraid of leaving something blank, 2 points is better than 1 point. When you get bad grades, pick your head up and keep moving forward. Don't be afraid to go to office hours to get explanations for all the homework problems you didn't solve. Don't let how difficult the problems are get to you, just throw yourself into it instead of getting scared off.*

## Other comments

*First and foremost, starting homework early is extremely important. Sleep early the night before lecture, otherwise it'll be hard to stay awake the next day and really easy to fall behind in the course. Go to the professors' OH after lecture. Go to the extra help sessions and OH on the weekends to ask for help in clarifying and understanding lecture material or why you got points off. Practice writing out your proofs before the hw sessions as well...I regretted not doing that enough. In general, the course takes a lot of time, so you need to be sure to allocate enough time to work on it to get as much as you want out of it.*

# Other comments

*Pay attention in lecture, don't think like “oh I'll just watch them later online”.*

*Pay attention in class, go to recitation, review the material every week, and go to office hours.*

*Go to recitation.*

# Other comments

*Think of it as a course that will give you a fantastic overview of CS theory – the ride will be tough, but try to focus less on the grades and more on enjoying understanding the material.*