

**15-251**

# **Great Theoretical Ideas in Computer Science**

**Lecture I:  
Introduction to the course**

## **Instructors:**

**Bernhard Haeupler**



**Anil Ada**

*Jan 17th, 2017*

What is computer science?

What is *theoretical* computer science?

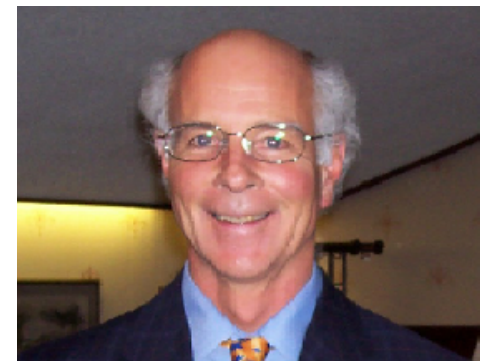
# Motivational Quote of the Course

“Computer Science is no more about computers than astronomy is about telescopes.”



*Edsger Dijkstra*

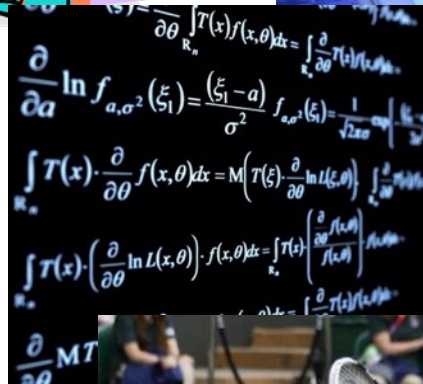
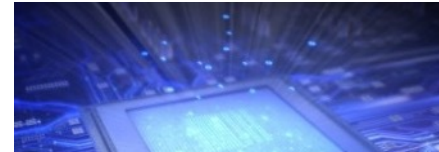
- *Michael Fellows*



# What is computer science?

Is it branch of:

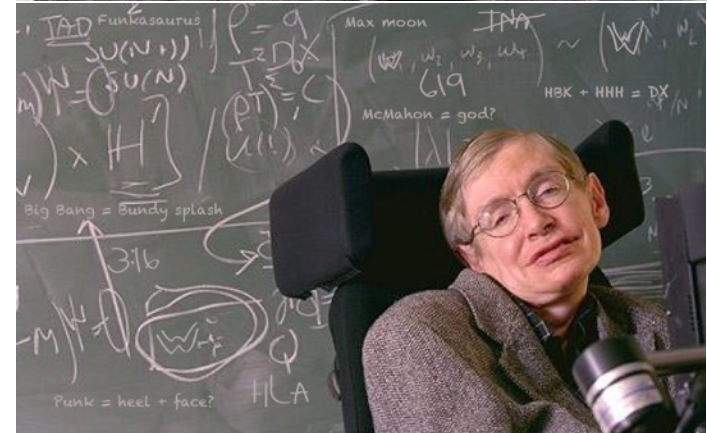
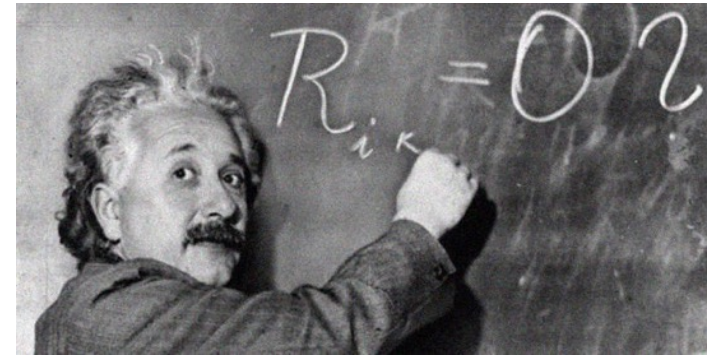
- science?
- engineering?
- math?
- philosophy?
- sports?



# Physics

## Theoretical physics

- come up with mathematical models
- Nature's language is mathematics**
- derive the logical consequences

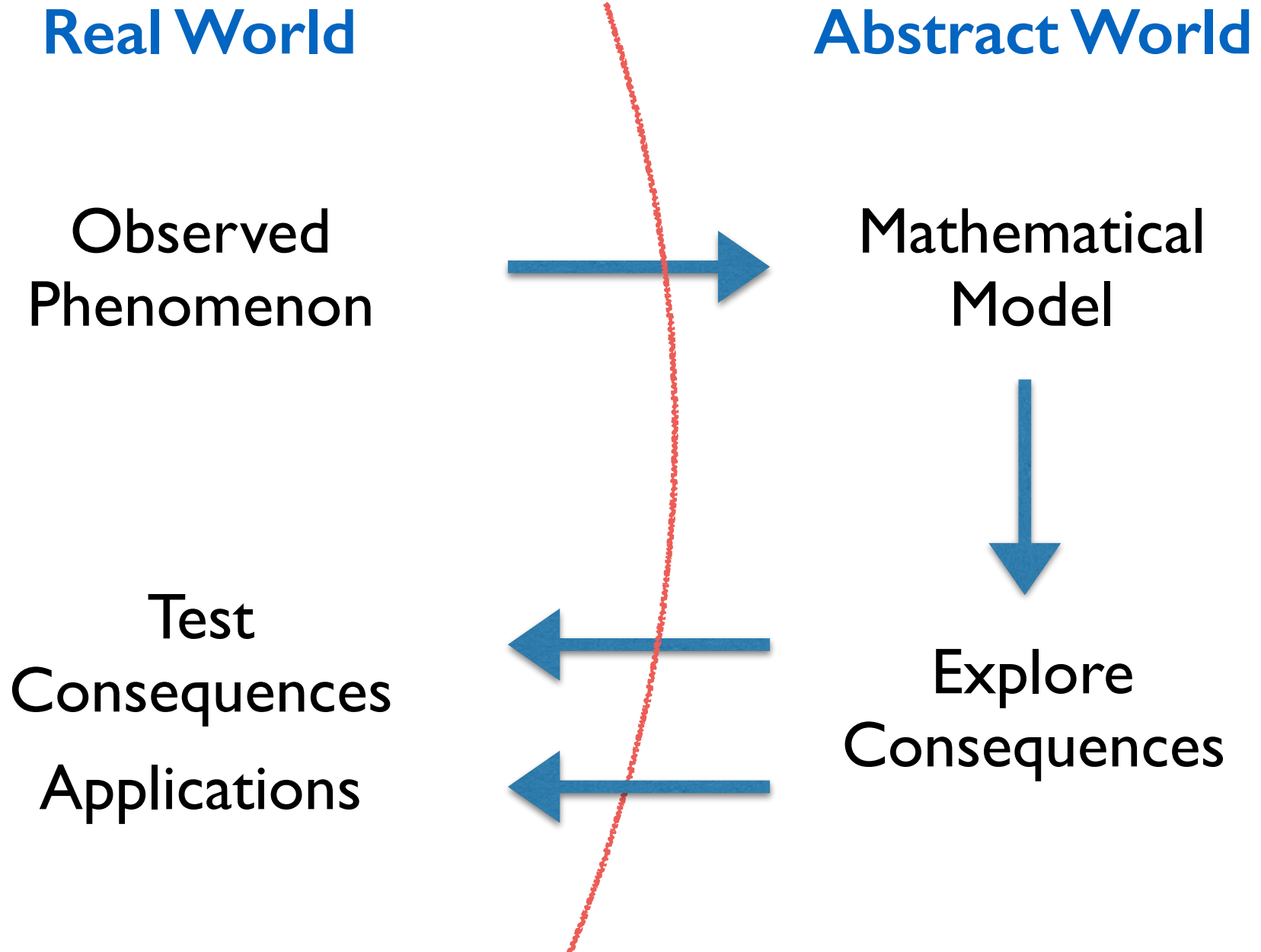


## Experimental physics

- make observations about the universe
- test mathematical models with experiments

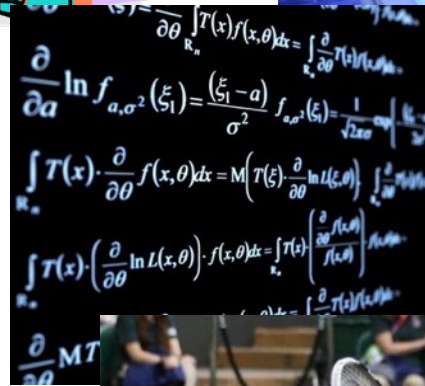
## Applications/Engineering

# The role of theoretical physics



# (Theoretical) Physics

- science?
- engineering?
- math?
- philosophy?
- sports?





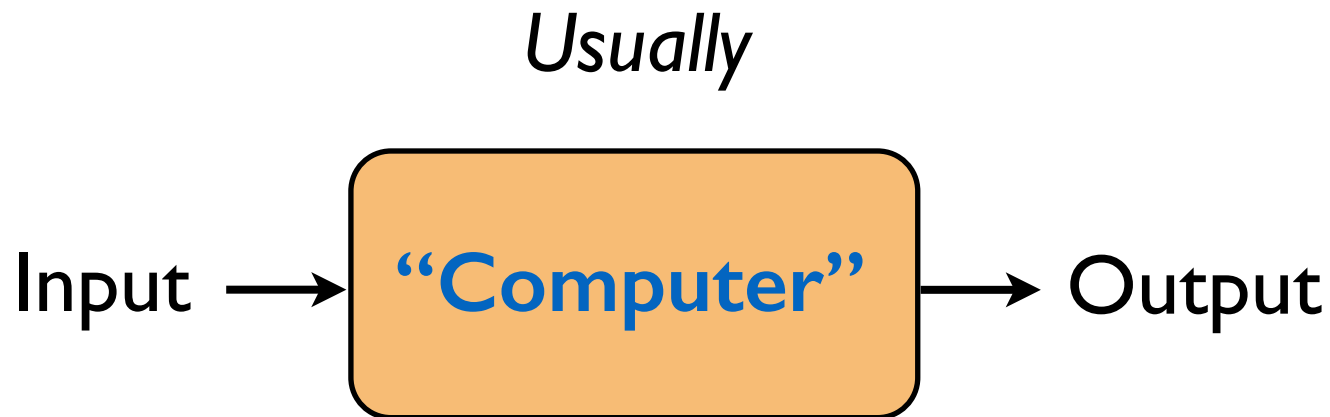
# Computer Science

The science that studies **computation**.

**Computation**: manipulation of information/data.

**Algorithm**: description of how the data is manipulated.

**Computational problem**: the input-output pairs.





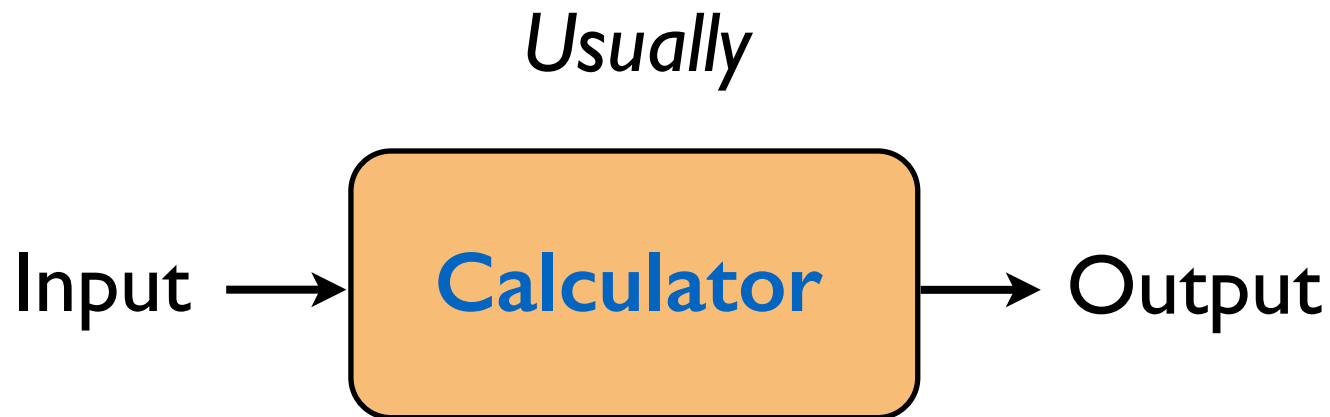
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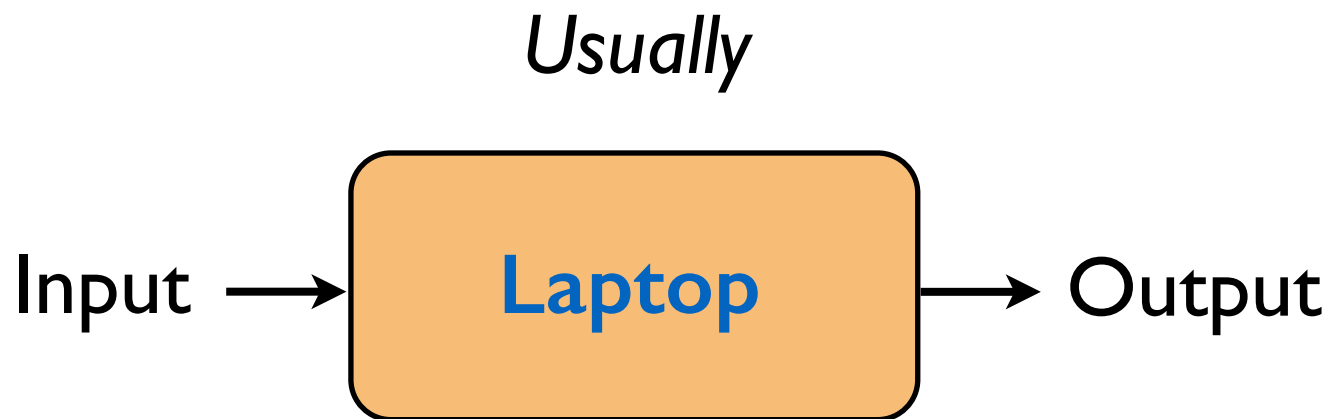
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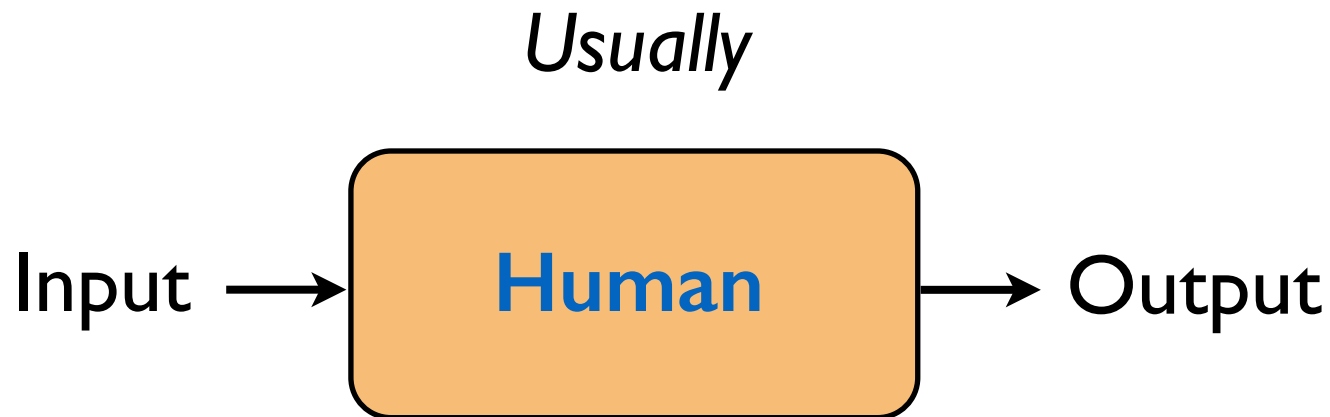
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# “Computers” in early 20th century



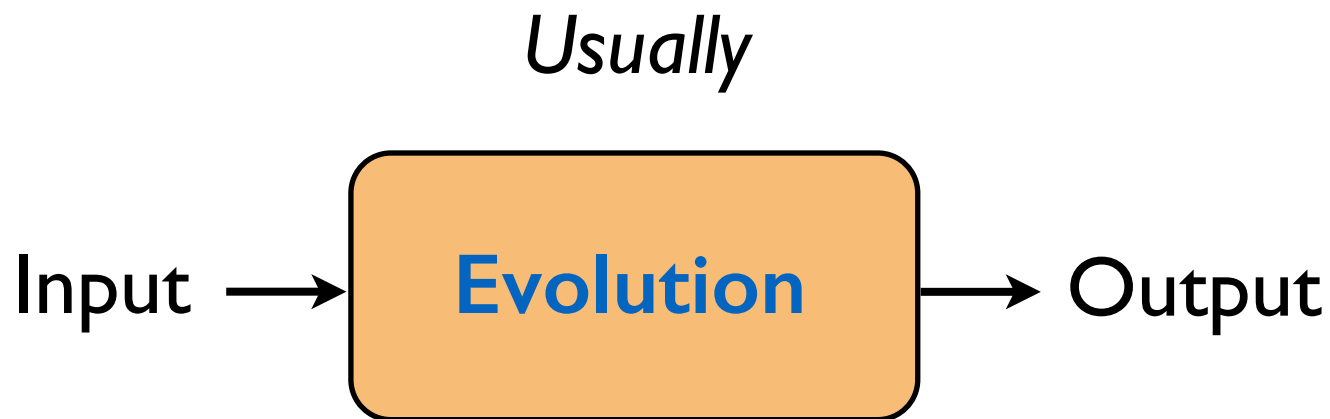
# Computer Science

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# The computational lens



Computational physics

Computational biology

Computational chemistry

Computational neuroscience

Computational economics

Computational finance

Computational linguistics

Computational statistics

...

# Wikipedia definition

“ Computer Science deals with the theoretical foundations of **information** and **computation**, together with practical techniques for the implementation and application of the foundations.”

- *Wikipedia*



# The role of theoretical computer science

Build a mathematical model for computation.

Explore the logical consequences.  
Gain insight about computation.

<http://youtu.be/pTeZP-XfuKI>

<https://goo.gl/gGkpMv>

<http://youtu.be/J4TkHuTmHsg>

Look for interesting applications.



CMU undergrad

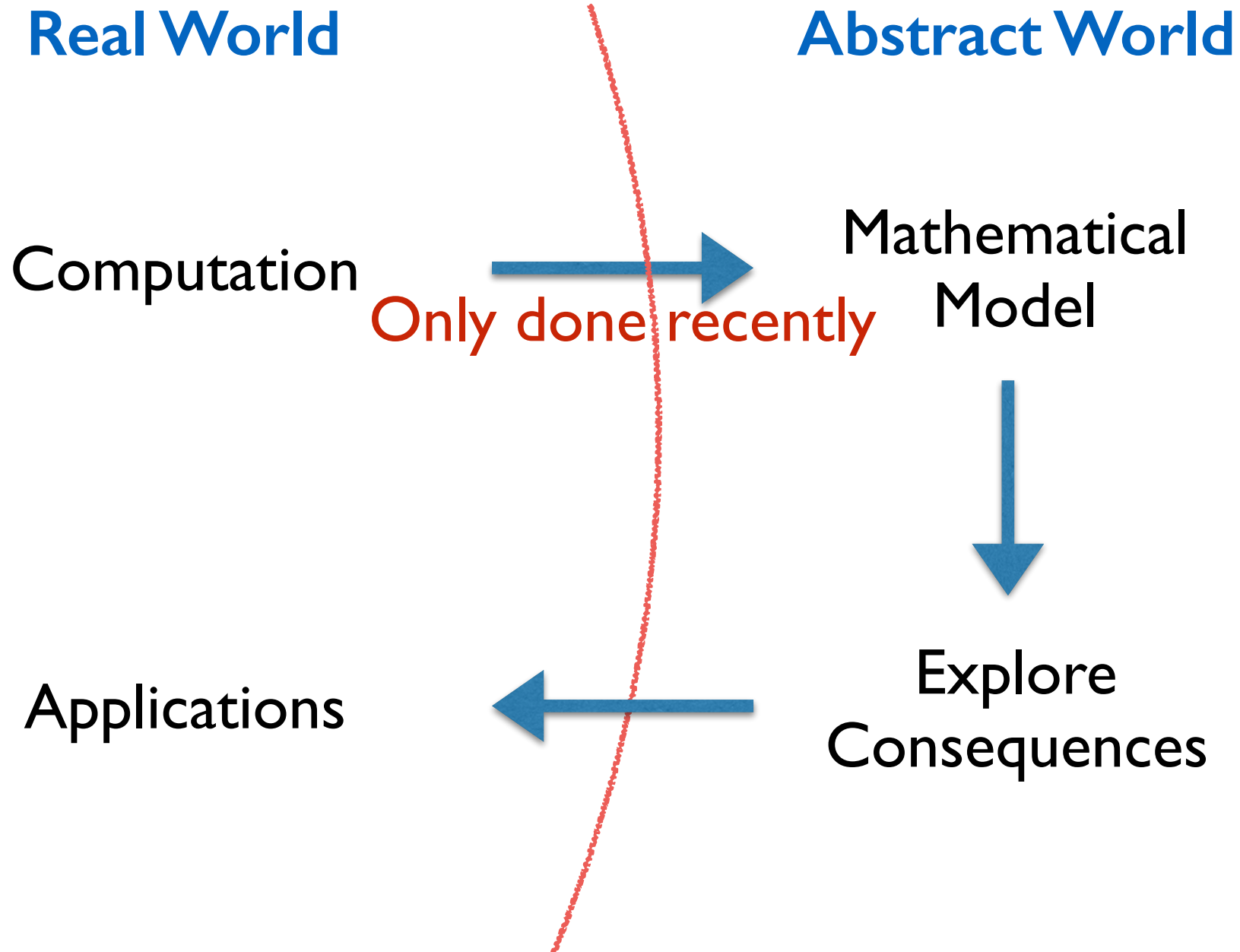


CMU Prof.



OK, we don't have  
everybody

# The role of theoretical computer science



# Simple examples of computation

We have been using algorithms for thousands of years.

$$\begin{array}{r} 5127 \\ \times 4265 \\ \hline 25635 \\ 307620 \\ 1025400 \\ 20508000 \\ \hline 21866655 \end{array}$$

# Simple examples of computation

We have been using algorithms for thousands of years.

Euclid's algorithm (~ 300BC):

```
def gcd(a, b):  
  
    while (a != b):  
  
        if (a > b):  
            a = a - b  
  
        else:  
            b = b - a  
  
    return a
```

# Formalizing computation

We have been using algorithms for thousands of years.

**Algorithm/Computation** was only **formalized** in the 20th century!

Someone had to ask the right question.

# David Hilbert, 1900



## The Problems of Mathematics

*“Who among us would not be happy to lift the veil behind which is hidden the future; to gaze at the coming developments of our science and at the secrets of its development in the centuries to come? What will be the ends toward which the spirit of future generations of mathematicians will tend? What methods, what new facts will the new century reveal in the vast and rich field of mathematical thought?”*

# 2 of Hilbert's Problems

## Hilbert's 10th problem (1900)

Is there a finitary procedure to determine if a given multivariate polynomial with integral coefficients has an integral solution?

e.g. 
$$5x^2yz^3 + 2xy + y - 99xyz^4 = 0$$

## Entscheidungsproblem (1928)

Is there a finitary procedure to determine the validity of a given logical expression?

e.g. 
$$\neg \exists x, y, z, n \in \mathbb{N} : (n \geq 3) \wedge (x^n + y^n = z^n)$$

(Mechanization of mathematics)



## 2 of Hilbert's Problems

**Fortunately**, the answer turned out to be NO.

# 2 of Hilbert's Problems

## Gödel (1934):

Discusses some ideas for mathematical definitions of computation. But not confident what is a good definition.



## Church (1936):

Invents [lambda calculus](#).

Claims it should be the definition of an “algorithm”.



## Gödel, Post (1936):

Arguments that Church's claim is not justified.

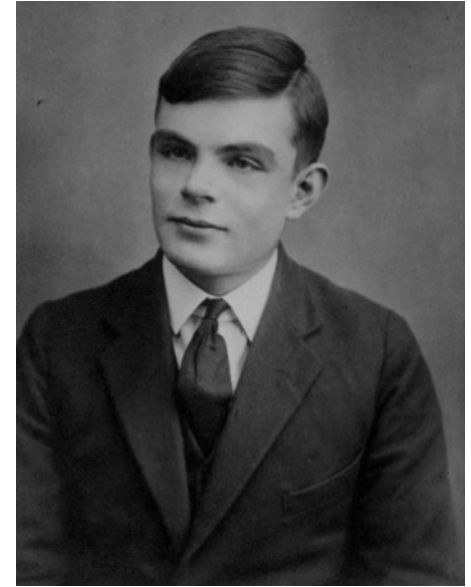


Meanwhile... in New Jersey... a certain British grad student, unaware of all these debates...

## 2 of Hilbert's Problems

**Alan Turing (1936, age 22):**

Describes a new model for computation,  
now known as the **Turing Machine**.™



**Gödel, Kleene, and even Church:**

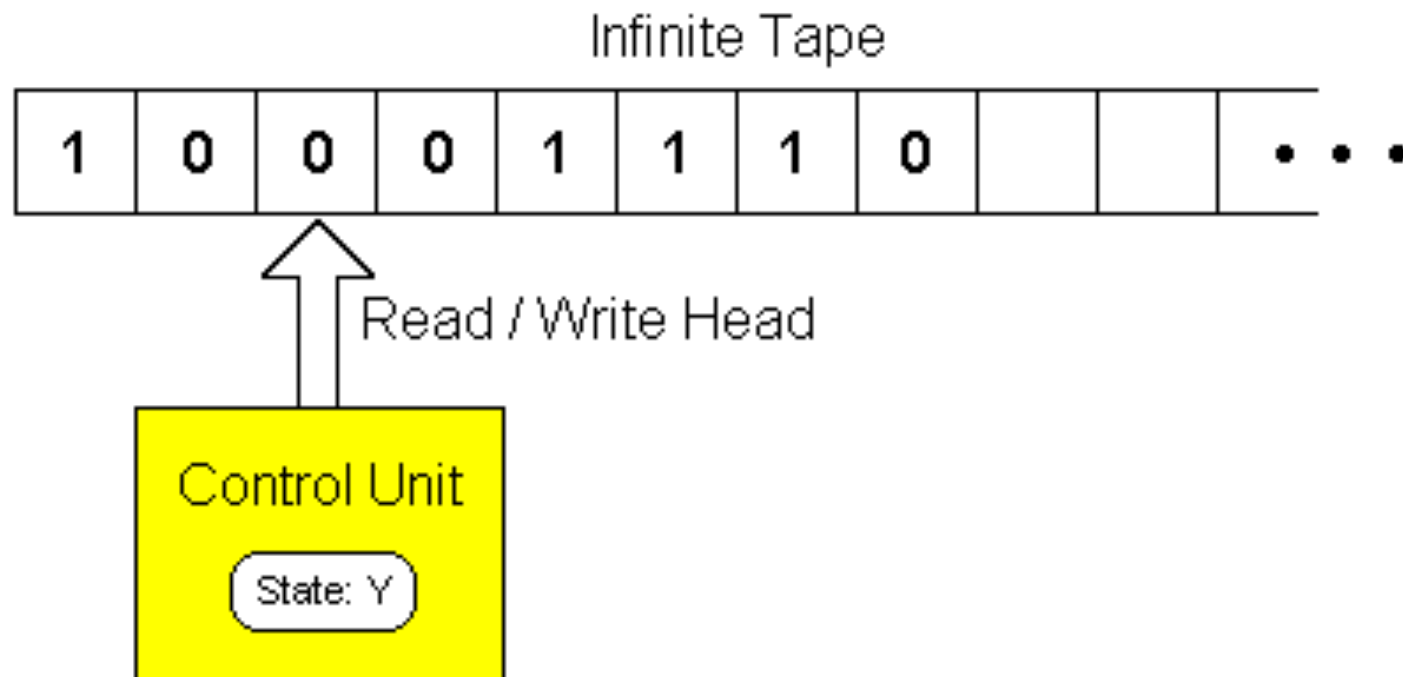
“Umm. Yeah. He nailed it. Game over. “Algorithm” defined.”

**Turing (1937):**

TMs  $\equiv$  lambda calculus

# Formalization of computation: Turing Machine

## Turing Machine:



# Church-Turing Thesis

## Church-Turing Thesis:

The intuitive notion of “computable” is captured by functions computable by a Turing Machine.

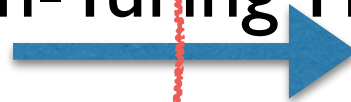
## (Physical) Church-Turing Thesis

Any computational problem that can be solved by a physical device, can be solved by a Turing Machine.

**Real World**

**Abstract World**

Church-Turing Thesis



# Back to Hilbert's Problems

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(Mechanization of mathematics)

# Back to Hilbert's Problems

## Hilbert's 10th problem (1900)

Is there **an algorithm (a TM)** to determine if a given multivariate polynomial with integral coefficients has an integral solution?

e.g. 
$$5x^2yz^3 + 2xy + y - 99xyz^4 = 0$$

## Entscheidungsproblem (1928)

Is there **an algorithm (a TM)** to determine the validity of a given logical expression?

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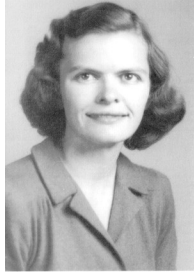
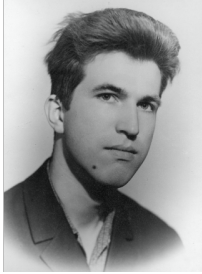
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# Back to Hilbert's Problems

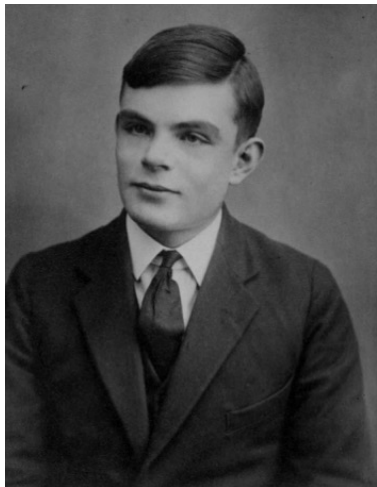
## Hilbert's 10th problem (1900)

**Matiyasevich-Robinson-Davis-Putnam (1970):**



There is no algorithm to solve this problem.

## Entscheidungsproblem (1928)

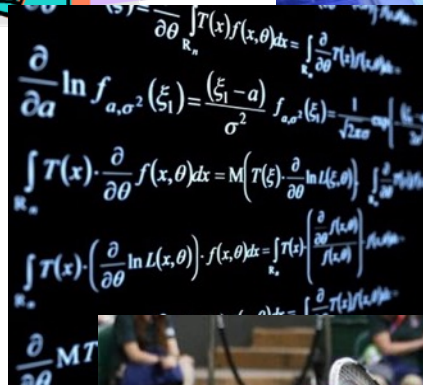
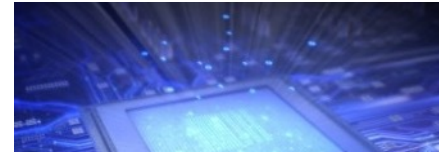


**Turing (1936):**

There is no algorithm to solve this problem.

# Computer science

- science?
- engineering?
- math?
- philosophy?
- sports?



# 2 Main Questions in TCS

**Computability** of a problem:

Is there an algorithm to solve it?

**Complexity** of a problem:

Is there an **efficient** algorithm to solve it?

- time
- space (memory)
- randomness
- quantum resources

# Computational Complexity

**Complexity** of a problem:

Is there an **efficient** algorithm to solve it?

- time
- space (memory)
- randomness
- quantum resources

**2 camps:**

- trying to come up with efficient algorithms  
(algorithm designers)
- trying to show no efficient algorithm exists  
(complexity theorists)

# Computational Complexity

## 2 camps:

- trying to come up with efficient algorithms  
(algorithm designers)
- trying to show no efficient algorithm exists  
(complexity theorists)

multiplying two integers

factoring integers

sorting a list

protein structure prediction

simulation of quantum systems

computing Nash Equilibria of games

# Some other interesting questions

If a problem has a space-efficient solution does it also have a time-efficient solution?

Can every randomized algorithm be derandomized efficiently?

Can we use quantum properties of matter to build faster computers?

**P vs NP**

# Learning Objectives



# Topics Overview

**Part 1**: Formalizing the notions of **problems**, **algorithms**, and **computability**.

**Part 2**: Efficient computation:  
basic algorithms and complexity

**Part 3**: Some highlights of theoretical CS and the mathematics behind them.



# This is a “big picture” course

Finite automata

Turing machines

Uncountability and Undecidability

Graph theory

Time Complexity

NP-completeness

Gödel's Incompleteness Theorems

Approximation algorithms

Probability

Random Walks

Randomized algorithms

Cryptography

Basic number theory

Quantum Computation



# Goals

- Learn about the **foundational ideas and concepts** in the **theory of computation**.
- Learn the **mathematical constructs** and **techniques** needed to understand and develop key computational concepts.
- Improve **rigorous, logical**, and **abstract** thinking skills.
- Develop **problem-solving skills**.
- Refine **proof-writing skills**.
- Express complex ideas and arguments **clearly**, both in written and oral form.
- **Cooperate** with others in order to solve challenging and rigorous problems related to the study of computer science.

# **A review of the course syllabus**

# Webpage

**Course webpage: [www.cs.cmu.edu/~1525/](http://www.cs.cmu.edu/~1525/)**

# Grading Scheme

## Grading:

12 homework assignments

30%

2 midterm exams

20% + 20% = 40%

*Mar 1, Apr 19*

*6:30pm - 9:30pm*

1 final exam

25%

Participation (attending classes and recitations)

5%

# Grading Scheme

## Alternate Grading:

12 homework assignments

30%, lowest 4 homeworks half-weighted

2 midterm exams

Higher midterm: 30%

1 final exam

35%

Participation (attending classes and recitations)

5%

## Important Note:

With this scheme, max letter grade you can receive is a C.

# A poll

What is your favorite TV show?

- Game of Thrones
- Breaking Bad
- Seinfeld
- Friends
- The Wire
- Sherlock
- The Sopranos
- Arrested Development
- Sesame Street
- None of the above
- I don't watch TV!



# Homework System

**4 types of questions:**

**SOLO, GROUP, OPEN COLLABORATION,  
PROGRAMMING**

**SOLO - work by yourself**

**GROUP - work in groups of 3 or 4**

**OPEN - work with anyone you would like from class**

**PROG - same rules as SOLO. submit to Autolab.**

# Homework System

## General rules:

Don't share written material with anyone.

Erase public whiteboard when done.

Can search books to learn more about a subject.

**Can't** Google specific keywords from the homework.

Always cite your sources!

Think about a problem before you collaborate.

# Homework System

## Homework writing sessions:

Wednesdays 6:30pm to 7:50pm at DH 2210

Write the solutions to a random subset of the problems.

You must practice writing the solutions beforehand!!!

You will lose points for poor presentation.

You get 20% of the credit for the question if you write:

- nothing
- “I don’t know”, or
- “WTF!”

# Homework System

## Homework Grading:

### Step 1:

TAs grade and give back the hw on Friday.

You will know who graded which question.

### Step 2:

If

- you think there has been a mistake in grading
- you don't understand why you lost points

email the TA who graded the question.

(attach a picture of your write-up)

# Homework System

**Learn from your mistakes —> more points:**

**To get back 25% of the lost credit on a problem:**

**Submit electronically (via email) a completely correct solution by 10pm Sunday.**

# Piazza

Everyone must sign up.

Course announcements will be made on Piazza.  
You have to check it every day.

Great resource, make use of it.

Please be polite.

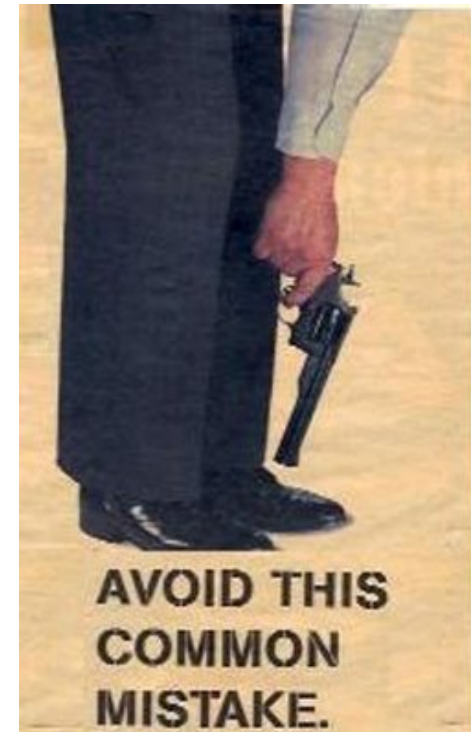


Don't give away any hints.

# Office hours

See course webpage.

You have to use the OHs!



We have separate **conceptual OHs**.

# Small Group Review Sessions

Sign up for an hour long time slot.

Review material with a TA and 2, 3 other students.



# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Lecture I

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Lecture 1.5 (6:30 - 7:50pm) (for this week only)

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Lecture 2

Office hour (Anil) (this week OHs start on Thu)

Review that week's material.

Homework comes out.

Maybe start thinking about SOLO problems.

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Recitation

Make progress on SOLO problems.

Start thinking about the GROUP problems.

Make appointments to meet with your group.

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Meet with your group.

Make some progress on the questions.

Maybe solve some of them.

Go to office hours.

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Meet with your group.

Go to office hours, get some help.

Solve some more problems.

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Finish up GROUP problems.

Go to office hours.

# A typical week

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Realize that you still need to do the OPEN problem(s)!

Express hate towards the professors.

Lecture

Rush to OH to get help.

Don't sleep until you solve the hardest problem.

I hate you this much





# A typical week

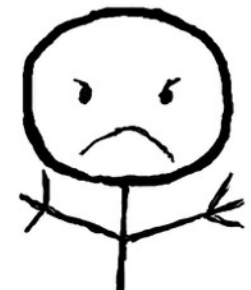
| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     |     |

Practice writing up the solutions to the problems.

Realize you have a mistake in one of the questions.



I hate you this much



Express hate towards the professors.

*Learning moment:*

write solution down once you think you figured it out.

# Keys to success in this course

**Come to lecture tomorrow!!**

# Video