Homework 10
due April 27th in class

1. Let $G = (V, E)$ be any graph and let M be any matching in G. A *subgraph H* of G is any graph whose set of vertices and edges are a subset of the set of vertices and edges of G respectively. Show that G contains a subgraph H, where H is bipartite and contains at least $\frac{1}{2}(|E(G)| + |M|)$ edges, where $|E(G)|$ is the number of edges in G, and $|M|$ is the number of edges in the matching M.

2. A *hospital cover* of a graph $G = (V, E)$ is a subset of the vertices $H \subseteq V$ such that every $v \in V \backslash H$ has at least one neighbor in H. Fix some $d > 1$. Show that if every vertex in G has degree at least d, then there is a hospital cover of G of size at most $|V| \frac{1 + \ln(d+1)}{d+1}$.

   Hint: This has similar flavor to showing that an independent set of a certain size must exist (which we proved in class).