15-252

Assignment 3 — Due: February 9, 2018.

1 Blow-Up (100)

Background

Suppose $G = \langle V, E \rangle$ is a (2,2)-regular directed graph: every edge has indegree 2 and outdegree 2. A labeling of G over alphabet $\Sigma = \{a, b\}$ is a permutation labeling if every node has exactly one in/out-transition labeled s for $s \in \Sigma$.

All automata below are assumed to have initial state set $I = Q$.

Task

A. Find a way to count permutation labelings in a (2,2)-regular graph. In particular, how many are there for a de Bruijn graph $B(k)$ and how many are there for a circulant $C(n; s, t)$?

B. Consider any permutation labeling on an automaton built on $C(n; 0, 1)$. Flip the label of a transition at a self-loop. Show that full exponential blow-up occurs.

C. Again consider any permutation labeling on an automaton built on $C(n; 0, 1)$. Flip the label of a stride-1 transition (see below). What happens?