15-252

Assignment 3 Due: February 23, 2018.

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1 Kolmogorov versus Palindromes (50)

Background

Suppose M is a one-tape Turing machine recognizing palindromes over $\{0, 1\}$. We say that M crosses tape cell number i if either

- the head moves right from i to $i + 1$, or
- the head moves left from $i + 1$ to i.

We can construct of a crossing sequence $((p_1, s_1), (p_2, s_2), \ldots)$ of all crossings of position i keeping track of the state p_i and the read symbol s_i at the moment of crossing (before the move). Note that right/left crossings must alternate.

Write $T(x)$ for the running time of M on input x, and assume that the machine always halts with the head on the right end of the string (it starts on the left). To streamline the argument a bit, it's best to consider input of the form $x0^n x^{\mathsf{op}}$ where $|x| = n$. The region $[n + 1, n + 2, \ldots, 2n]$ is called the desert. Note that every position in the desert has at least one crossing.

Task

A. Show that some position I in the desert must have a crossing sequence of length $m \leq T(x)/n$.

B. Show that x is the unique string of length n such that input $x0^{I-n}$ produces this crossing sequence.

C. Exploit part (B) to give a compact description of x and conclude that we cannot have $T(x) = o(n^2)$.

2 Uninspired Sets (50)

Background

Let $C(x \mid y)$ be the conditionl Kolmogorov-Chaitin complexity of $x \in \mathbf{2}^\star$, given y. For any set $A \subseteq \mathbb{N}$ write $A_n = A \cap \{0, 1, \ldots, n-1\}$ for the initial segment of A of length n. Think of A_n as bitvector of length n.

As we have seen, incompressibility with respect to Kolmogorov-Chaitin complexity is akin to randomness: there are no particular patterns one could exploit to obtain a shorter definition. How about the opposite notion? Call $A \subseteq \mathbb{N}$ uninspired if there is a constant c such that

$$C(A_n \mid n) \leq \log n + c.$$

So only some $\log n$ bits are needed to describe the corresponding bitvector of length n.

Task

A. Show that any decidable set A is uninspired.

B. How about the Halting Set K? State whether K is uninspired and explain your reasoning.

C. How about the complement of the Halting Set? Again, state whether this set is uninspired and explain your reasoning.

Comment

The intuitive version of Kolmogorov-Chaitin is good enough for this application, you don't have to worry about prefix programs.