CMU 15-252 **Spring 2018**

HOMEWORK 8
DUE APRIL 6

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1. VC-dimension (named after Vapnik and Chervonenkis) is an important concept in many areas, in particular in Learning Theory. It gives information about the difficulty of learning a given concept. Given a collection $\mathcal{S} = \{S_1, \ldots, S_m\}$ of subsets of a finite set U, the VC-dimension of $\mathcal{S}$ is the size of the largest set $X \subseteq U$ such that for every $X' \subseteq X$, there is an i for which $S_i \cap X = X'$ (we say that X is shattered by $\mathcal{S}$).

   We say that a boolean circuit C succinctly represents a collection $\mathcal{S}$ of subsets of U if the following holds: for $i \in \{1, 2, \ldots, m\}$ and $x \in U$, $C(\langle i, x\rangle) = 1$ if and only if $x \in S_i$.

   Finally, the language VC-DIMENSION is the set strings $\langle C, k\rangle$ such that C succinctly represents a collection of subsets $\mathcal{S}$ whose VC-dimension is at least k.

   What is the smallest class in the polynomial hierarchy in which you can put VC-DIMENSION? Prove your answer.

2. The complexity class PSPACE denotes the set of languages that can be decided by a TM M with the property that there exists a polynomial p such that for any input $x \in \Sigma^*$, $M(x)$ accesses at most $p(|x|)$ cells of its tape. Prove that PSPACE $\subseteq$ EXP.