15-251: Great Theoretical Ideas In Computer Science

Recitation 2

Regular Announcements

- Homework solution sessions: Saturday and Sunday, 12:30-1:30, GHC 4215
- Homework resubmissions next Friday - check the course website for details
- Come talk to us if you had difficulties on Homework 1!

Definitions For All

- **Deterministic Finite Automaton (DFA)**: A DFA M is a machine that reads a finite input one character at a time in one pass, transition from state to state, and ultimately accepts or rejects. Formally, M is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where
  - Q is a **finite**, **non-empty** set of states
  - Σ is the **finite**, **non-empty** alphabet
  - $\delta : Q \times \Sigma \to Q$ is the transition function
  - $q_0 \in Q$ is the starting state
  - $F \subseteq Q$ is the set of accepting states
- **Regular language**: A language L is regular if $L = L(M)$ for some DFA M (M decides L).
- We have shown that if L_1 and L_2 are both regular languages over Σ^*, for some fixed Σ, then the following are all regular.
  - $\overline{L_1}$
  - $L_1 \cup L_2$
  - $L_1 \cap L_2$
  - $L_1 L_2$ (the concatenation of two regular languages)

Odd Ones Out

Draw a DFA that decides the language

$$L = \{x : x \text{ has an even number of 1s and an odd number of 0s}\}$$

over the alphabet $\Sigma = \{0, 1\}$.

Adam, I'm Ada!

Show that, if $|\Sigma| > 1$, then

$$L = \{x \mid x \in \Sigma^* \text{ and } x = x^r\}$$

is an irregular language.

Suffering with Suffixes

Given a word w, we say that u is a proper suffix of w if there is $v \neq \epsilon$ such that $w = vu$. For a language L, define

$$\mathsf{SUFF}(L) = \{w \in L : \text{no proper suffix of } w \text{ is in } L\}.$$

Show that if L is regular, then so is $\mathsf{SUFF}(L)$, as follows. Give an exact description of a DFA recognizing $\mathsf{SUFF}(L)$, explicitly stating how Q, δ, q_0 and F are defined. Furthermore, briefly explain the reasoning behind your construction. A formal proof of correctness is not needed.

Multiple Multiples (Extra Problem)

Let $\Sigma = \{0, 1\}$. For each $n \geq 1$, define

$$C_n = \{x \in \Sigma^* \mid x \text{ is a binary number that is a multiple of } n\}.$$

Show that C_n is regular for all n.

States For Days (Extra Problem)

For any $n \geq 1$, let

$$\mathcal{R}_n = \{x \mid x \in \{0,1\}^* \text{ and the } n\text{-th symbol from the right is a } 1\}.$$

Show that any DFA that accepts $\mathcal{R}_n$ must have at least 2^n states.