15-251: Great Theoretical Ideas In Computer Science

Recitation 6

Announcements

- Midterm 1 next **Wednesday, February 28**! It will be held in **DH 2210** from **6.30pm to 9.30pm** in place of the writing session. (Note the later end time.)
- We will be holding topical reviews with the venues and times to be confirmed. Watch Piazza for updates!

Recap of some definitions and facts

- Stable Matching Problem
- Gale-Shapley Algorithm
- Regular graphs
- The tree-nity (the three salient features of a tree)
- Hamiltonian cycle
- The handshake lemma

Soulmates

Call a man m and a woman w "soulmates" if they are paired with each other in every stable matching.

(a) Given a man m and a woman w, design a polynomial-time algorithm to determine if they are soulmates.

(b) Give a polynomial time algorithm to determine if an instance of the stable matching problem has a *unique* stable matching.

"Clearly" Correct

A connected graph with no cycles is a tree. Consider the following claim and its proof.

> **Claim:** Any graph with n vertices and $n-1$ edges is a tree.
> **Proof:** We prove the claim by induction. The claim is clearly true for $n = 1$ and $n = 2$. Now suppose the claim holds for $n = k$. We'll prove that it also holds for $n = k + 1$. Let G be a graph with k vertices and $k - 1$ edges. By the induction hypothesis, G is a tree (and therefore clearly connected). Add a new vertex v to G by connecting it with any other vertex in G. So we create a new graph G' with $k+1$ vertices and k edges. The new vertex we added is clearly a leaf, so it clearly does not create a cycle. Also, since G was connected, G' is clearly also connected. A connected graph with no cycles is a tree, so G' is also a tree. So the claim follows by induction.

Explain why the given proof is incorrect.

2 3 Proofs 4 You

Give two proofs (one using induction and another using a degree counting argument) for the following claim: the number of leaves in a tree with $n \geq 2$ vertices is

$$2 + \sum_{\substack{v \in V \\ \deg(v) \geq 3}} (\deg(v) - 2)$$