

# I 5-25 I

## Great Ideas in Theoretical Computer Science

### Lecture I.5: On proofs + How to succeed in 25 I

*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E} [f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\hat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\hat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

□



January 17th, 2018

# Piazza poll

What is your favorite TV show?



# **PART I**

On proofs

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1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

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1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

# Is this a legit proof?

## **Proposition:**

Start with any number.

If the number is even, divide it by 2.

If it is odd, multiply it by 3 and add 1.

If you repeat this process, it will lead you to 4, 2, 1.

## **Proof:**

Many people have tried this, and no one came up with a counter-example.



# Is this a legit proof?

## ~~Proposition:~~ **Collatz Conjecture:**

Start with any number.

If the number is even, divide it by 2.

If it is odd, multiply it by 3 and add 1.

If you repeat this process, it will lead you to 4, 2, 1.

## **Proof:**

Many people have tried this, and no one came up with a counter-example.





# Is this a legit proof?

## Proposition:

$313(x^3 + y^3) = z^3$  has no solution for  $x, y, z \in \mathbb{Z}^+$ .

## Proof:

Using a computer, we verified that there is no solution for numbers with  $< 500$  digits.



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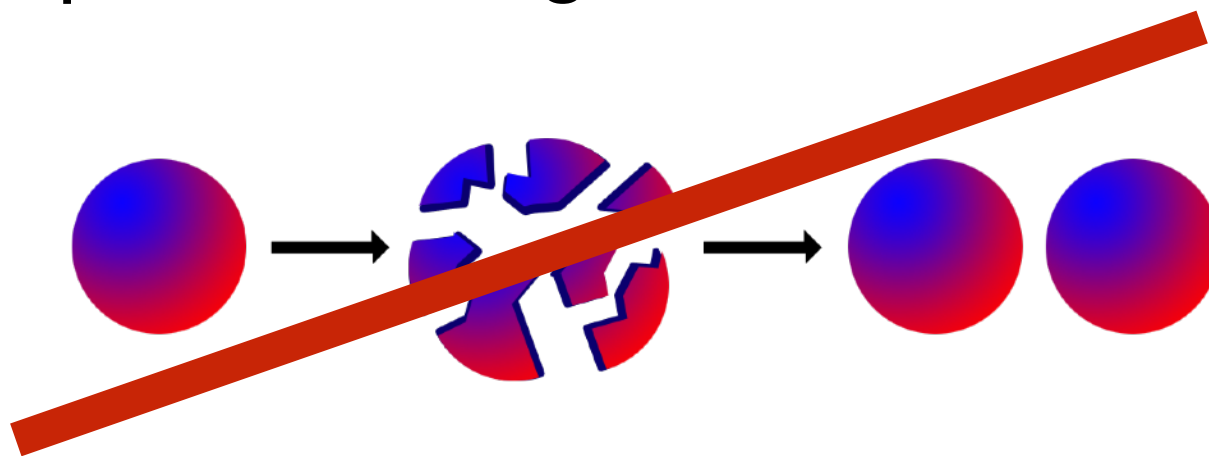
Using a computer, we verified that there is no solution for numbers with  $< 500$  digits.



# Is this a legit proof?

## Proposition:

Given a solid ball in 3 dimensional space, there is no way to decompose it into a finite number of disjoint subsets, which can be put together to form two identical copies of the original ball.



## Proof:

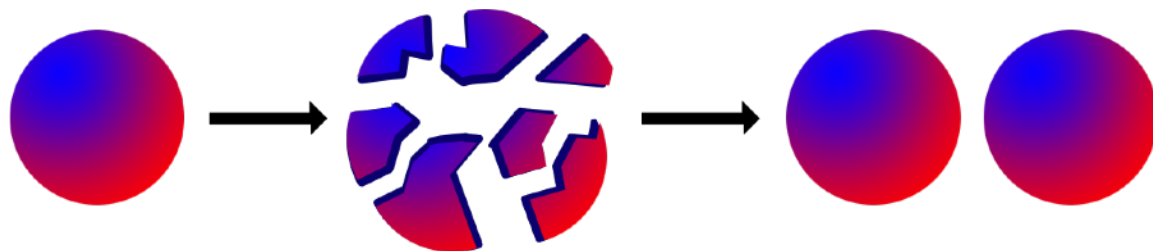
Obvious.



# Is this a legit proof?

## Banach-Tarski Theorem:

Given a solid ball in 3 dimensional space, there **is a** way to decompose it into a finite number of disjoint subsets, which can be put together to form two identical copies of the original ball.



## Proof:

Uses group theory... The pieces are such weird scatterings of points that they have no meaningful “volume”...

# Is this a legit proof?

**Proposition:**

$$1 + 1 = 2$$

**Proof:**

This is obvious?

# Is this a legit proof?

**Proposition:**

$$1 + 1 = 2$$

**Proof:**

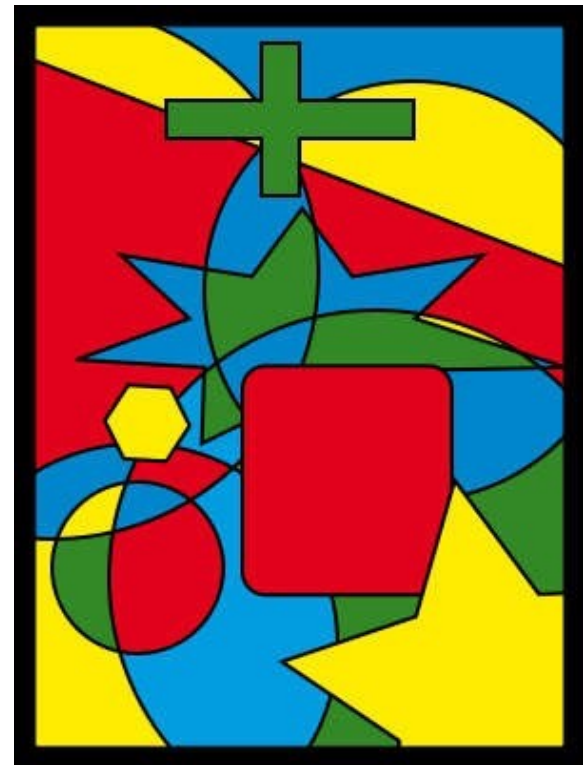
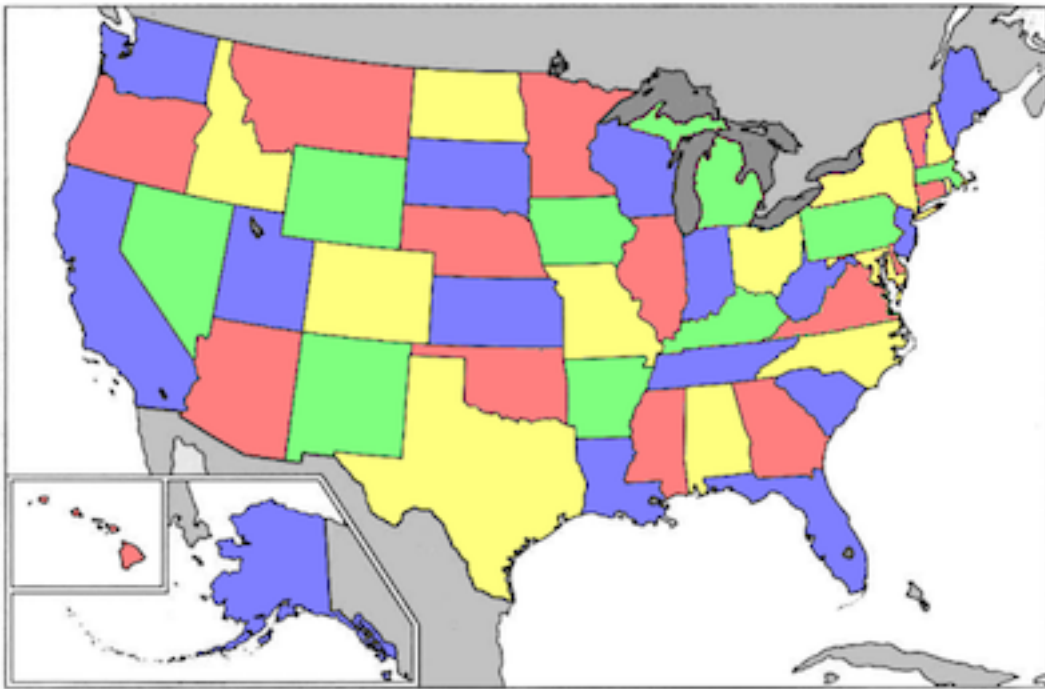
This is obvious!



# The story of 4 color theorem

## 1852 Conjecture:

Any 2-d map of regions can be colored with 4 colors so that no adjacent regions get the same color.



# The story of 4 color theorem

**1879:** Proved by **Kempe** in *American Journal of Mathematics*  
(was widely acclaimed)

**1880:** Alternate proof by **Tait** in *Trans. Roy. Soc. Edinburgh*

**1890:** **Heawood** finds a bug in **Kempe's** proof

**1891:** **Petersen** finds a bug in **Tait's** proof

**1969:** **Heesch** showed the theorem could in principle be reduced to checking a large number of cases.

**1976:** **Appel** and **Haken** wrote a massive amount of code to compute and then check 1936 cases.  
(1200 hours of computer time)





# The story of 4 color theorem

Much controversy at the time. Is this a proof?

What do you think?

Arguments against:

- no human could ever hand-check the cases
- maybe there is a bug in the code
- maybe there is a bug in the compiler
- maybe there is a bug in the hardware
- no “insight” is derived

**1997:** Simpler computer proof by  
Robertson, Sanders, Seymour, Thomas

# What is a mathematical proof?

*inference rules like* 
$$\frac{P, P \implies Q}{Q}$$

A mathematical **proof** of a **proposition** is a chain of **logical deductions** starting from a set of **axioms** and leading to the **proposition**.

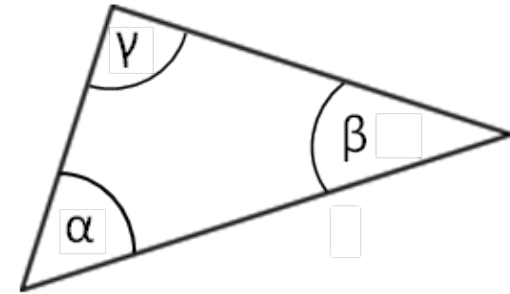
*propositions accepted to be true*

*a statement that is true or false*

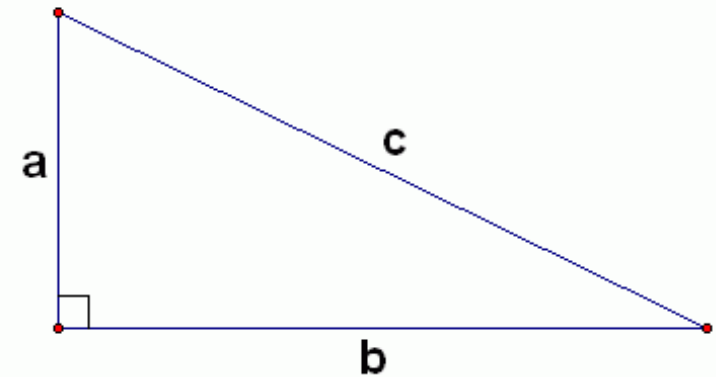


# Euclidian geometry

## Triangle Angle Sum Theorem

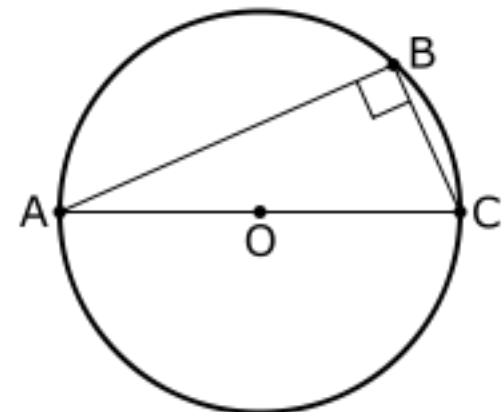


## Pythagorean Theorem



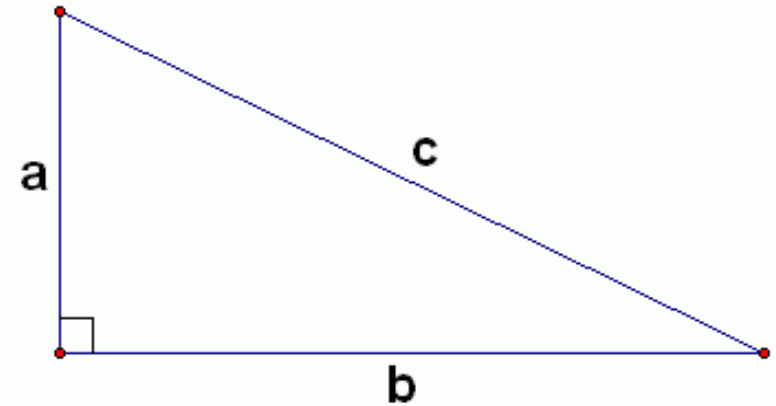
$$a^2 + b^2 = c^2$$

## Thales' Theorem



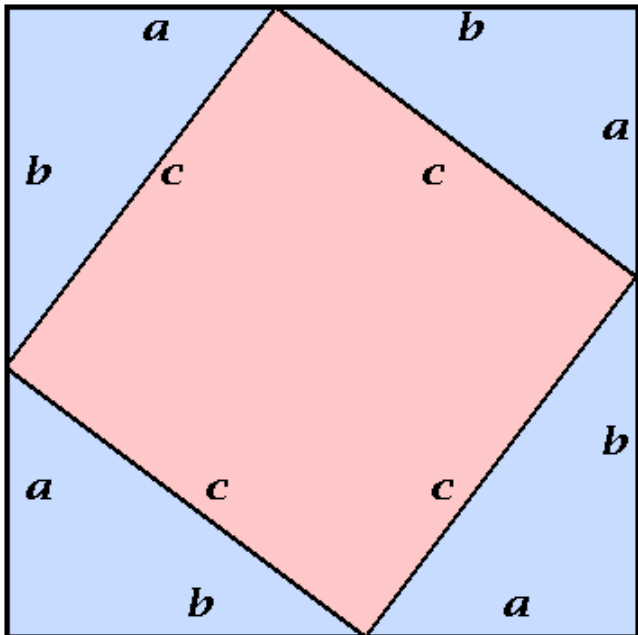
# Euclidian geometry

## Pythagorean Theorem



$$a^2 + b^2 = c^2$$

## Proof:



$$\begin{aligned}c^2 &= (a + b)^2 - 2ab \\ &= a^2 + b^2.\end{aligned}$$

Looks legit.



# Proof that square-root(2) is irrational

1. Suppose  $\sqrt{2}$  is rational.

Then we can find  $a, b \in \mathbb{N}$  such that  $\sqrt{2} = a/b$ .

2. If  $\sqrt{2} = a/b$  then  $\sqrt{2} = r/s$ ,  
where  $r$  and  $s$  are not both even.

3. If  $\sqrt{2} = r/s$  then  $2 = r^2/s^2$ .

4. If  $2 = r^2/s^2$  then  $2s^2 = r^2$ .

5. If  $2s^2 = r^2$  then  $r^2$  is even, which means  $r$  is even.

6. If  $r$  is even,  $r = 2t$  for some  $t \in \mathbb{N}$ .

7. If  $2s^2 = r^2$  and  $r = 2t$  then  $2s^2 = 4t^2$  and so  $s^2 = 2t^2$ .

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# Proof that square-root(2) is irrational

5a.  $r^2$  is even. Suppose  $r$  is odd.

5b. So there is a number  $t$  such that  $r = 2t + 1$ .

5c. So  $r^2 = (2t + 1)^2 = 4t^2 + 4t + 1$ .

5d.  $4t^2 + 4t + 1 = 2(2t^2 + 2t) + 1$ , which is odd.

5e. So  $r^2$  is odd.

5f. Contradiction is reached.

Odd number means not a multiple of 2.

Is every number a multiple of 2 or one more than a multiple of 2?

# Proof that square-root(2) is irrational

5b1. Call a number  $r$  **good** if  $r = 2t$  or  $r = 2t + 1$  for some  $t$ .

If  $r = 2t$ ,  $r + 1 = 2t + 1$ .

If  $r = 2t + 1$ ,  $r + 1 = 2t + 2 = 2(t + 1)$ .

Either way,  $r + 1$  is also **good**.

5b2. 1 is **good** since  $1 = 0 + 1 = (0 \cdot 2) + 1$ .

5b3. Applying 5b1 repeatedly, 2, 3, 4, ... are all **good**.

# Proof that square-root(2) is irrational

## Axiom of induction:

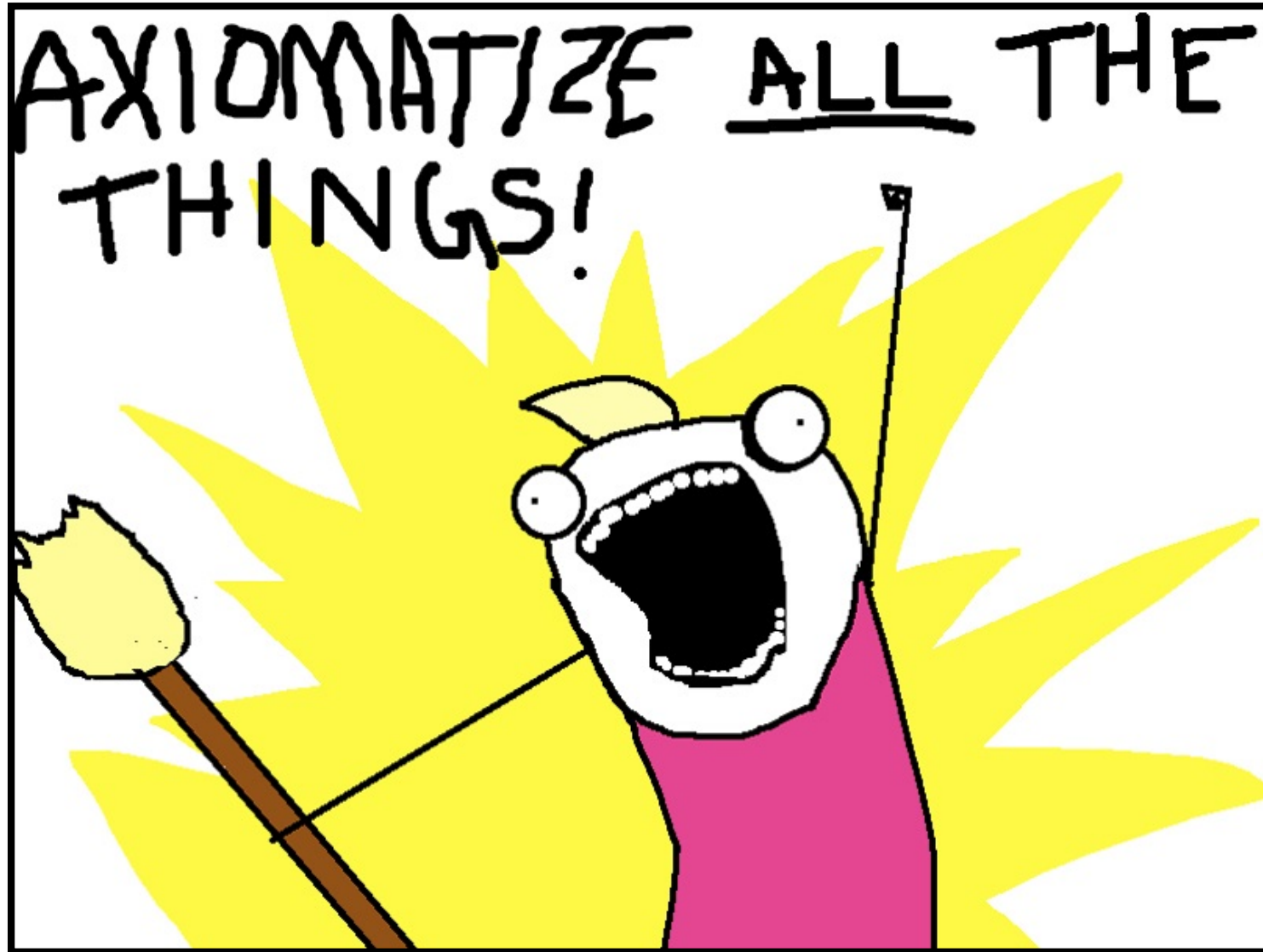
Suppose for every positive integer  $n$ , there is a statement  $S(n)$ .

If  $S(1)$  is true, and  $S(n) \implies S(n + 1)$  for any  $n$ ,  
then  $S(n)$  is true for every  $n$ .

Can every mathematical theorem be derived from a set of agreed upon axioms?

# Formalizing math proofs

A dream from late 19th and early 20th century.

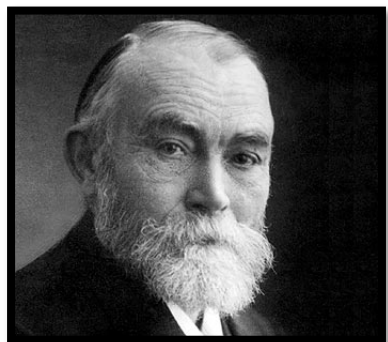


# Formalizing math proofs

After playing around, people realized you could seemingly do 100% of math using just the notions from set theory.

(Define natural numbers in terms of sets, ordered pairs in terms of sets, functions in terms of sets, sequences in terms of sets, real numbers, graphs, strings, automata, **everything** in terms of sets...)

# Formalizing math proofs



## Frege, 1893:

Proposes axioms for set theory.

Spends 10 years writing two thick books about the system.

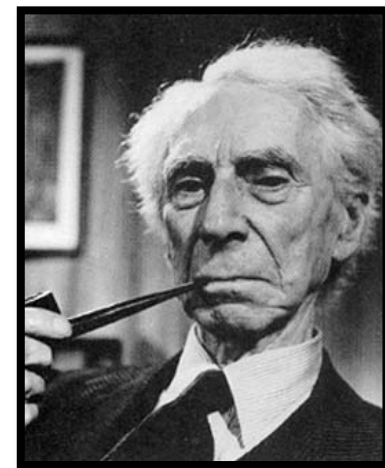
## Russell, 1903:

“Your axioms allow me to define  $D = \{x : x \notin x\}$ .

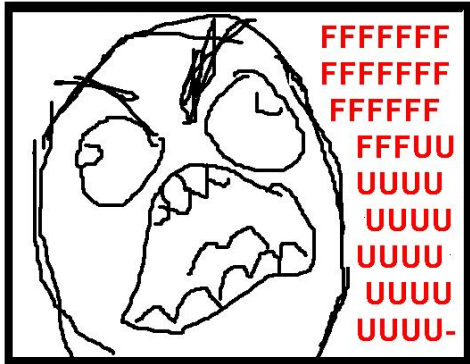
Now if  $D \in D$  then  $D \notin D$ .

And if  $D \notin D$  then  $D \in D$ .

Inconsistency, boom!”



# Formalizing math proofs



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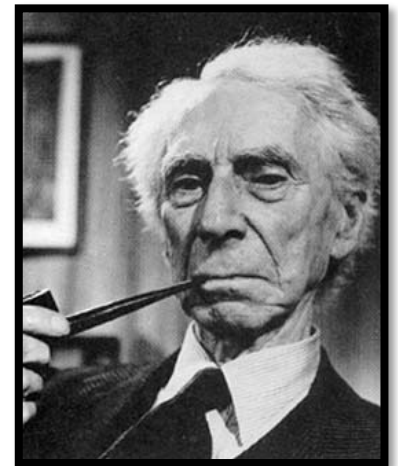
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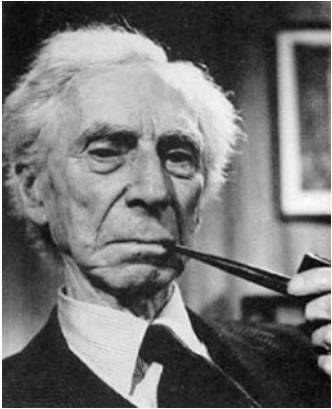
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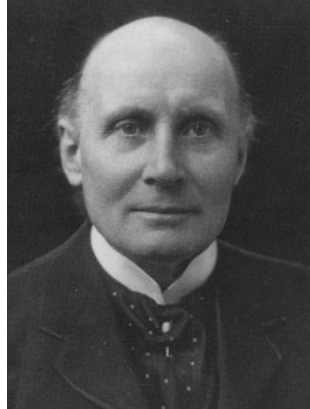


# Formalizing math proofs

## Principia Mathematica Volume 2



Russell



Whitehead

86 CARDINAL ARITHMETIC [PART III]

**\*110·632.**  $\vdash : \mu \in NC . \supset . \mu +_c 1 = \hat{\xi} \{ (\exists y) . y \in \xi . \xi - t'y \in sm''\mu \}$

*Dem.*

$\vdash . *110·631 . *51·211·22 . \supset$

$\vdash : Hp . \supset . \mu +_c 1 = \hat{\xi} \{ (\exists \gamma, y) . \gamma \in sm''\mu . y \in \xi . \gamma = \xi - t'y \}$

[\*13·195]  $= \hat{\xi} \{ (\exists y) . y \in \xi . \xi - t'y \in sm''\mu \} : \supset \vdash . Prop$

**\*110·64.**  $\vdash . 0 +_c 0 = 0$  [\*110·62]

**\*110·641.**  $\vdash . 1 +_c 0 = 0 +_c 1 = 1$  [\*110·51·61 . \*101·2]

**\*110·642.**  $\vdash . 2 +_c 0 = 0 +_c 2 = 2$  [\*110·51·61 . \*101·31]

**\*110·643.**  $\vdash . 1 +_c 1 = 2$

*Dem.*

$\vdash . *110·632 . *101·21·28 . \supset$

$\vdash . 1 +_c 1 = \hat{\xi} \{ (\exists y) . y \in \xi . \xi - t'y \in 1 \}$

[\*54·3]  $= 2 . \supset \vdash . Prop$

The above proposition is occasionally useful. It is used at least three times, in \*113·66 and \*120·123·472.

Writing a proof like this  
is like writing a computer program in machine language.

# LOGICOMIX



AN EPIC SEARCH FOR TRUTH

APOSTOLOS DOXIADIS AND CHRISTOS H. PAPADIMITRIOU

ART BY ALECOS PAPADATOS AND ANNIE DI DONNA

# Formalizing math proofs

It became generally agreed that you **could** rigorously formalize mathematical proofs.

But nobody wants to.  
(by hand, at least)

## **Interesting consequence:**

Proofs can be verified mechanically.

**One last story**



Lord Wacker von Wackenfels  
(1550 - 1619)

# Kepler Conjecture

**1611:**



**Kepler** as a New Year's present (!) for his patron, **Lord Wacker von Wackenfels**, wrote a paper with the following conjecture.

The densest way to pack oranges is like this:



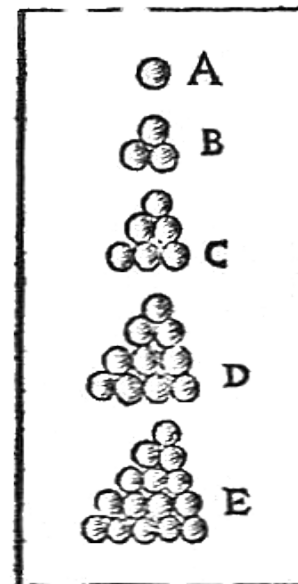
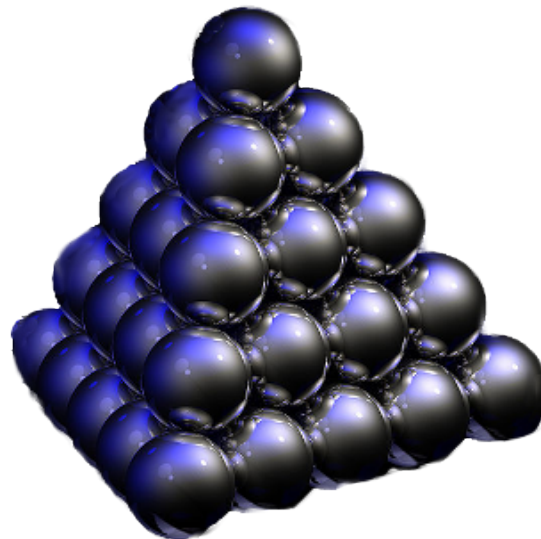
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# Kepler Conjecture

**2005:** Pittsburgher **Tom Hales** submits a 120 page proof in *Annals of Mathematics*.

Plus code to solve 100,000 distinct optimization problems, taking 2000 hours computer time.



*Annals* recruited a team of 20 refs.  
They worked for 4 years.  
Some quit. Some retired. One died.  
In the end, they gave up.

They said they were “99% sure” it was a proof.

# Kepler Conjecture



Hales: “I will code up a completely formal axiomatic deductive proof, checkable by a computer.”

**2004 - 2014:** Open source “Project Flyspeck”:

**2015:** Hales and 21 collaborators publish  
“A formal proof of the Kepler conjecture”.

# Formally proved theorems

Fundamental Theorem of Calculus (*Harrison*)

Fundamental Theorem of Algebra (*Milewski*)

Prime Number Theorem (*Avigad @ CMU, et al.*)

Gödel's Incompleteness Theorem (*Shankar*)

Jordan Curve Theorem (*Hales*)

Brouwer Fixed Point Theorem (*Harrison*)

Four Color Theorem (*Gonthier*)

Feit-Thompson Theorem (*Gonthier*)

Kepler Conjecture (*Hales++*)

## **Summary / Bottom Line**

In math, there are agreed upon rigorous rules for deduction. Proofs are either right or wrong.

Nevertheless, what constitutes an acceptable proof is a social construction.

(But computer science can help.)

# What does this all mean for 15-251?

A proof is an argument that can withstand all criticisms from a highly caffeinated adversary (your TA).



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which implies that

$$\widehat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

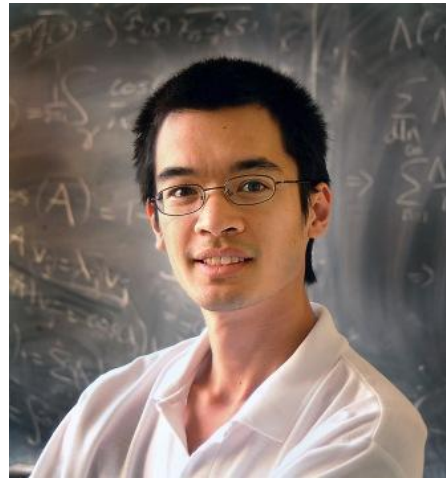
□

1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

# How do you find a proof?



No Eureka effect



Terence Tao

Fields Medalist,  
“MacArthur Genius”,

...

I don't have any magical ability. ... When I was a kid, I had a romanticized notion of mathematics, that hard problems were solved in 'Eureka' moments of inspiration. [But] with me, it's always, 'Let's try this. That gets me part of the way, or that doesn't work. Now let's try this. Oh, there's a little shortcut here.' You work on it long enough and you happen to make progress towards a hard problem by a back door at some point. At the end, it's usually, 'Oh, I've solved the problem.'

# How do you find a proof?

## Suggestions

Make 1% progress for 100 days.  
(Make 17% progress for 6 days.)

Understand the problem.

(List what is given to you. Write down what you need to derive.  
Unpack definitions.)

Figure out some meaningful special cases (e.g.  $n = 1$ ,  $n = 2$ ).

Simplify the problem.

Put yourself in the mind of the adversary.

(What are the worst-case examples/scenarios?)



# How do you find a proof?

## **Suggestions**

Look at proofs from notes, recitations.

Give breaks, let the unconscious brain do some work.

Develop good notation.

Use paper, draw pictures.

# How do you find a proof?

## Suggestions

Try different proof techniques.

- contrapositive  $P \implies Q \iff \neg Q \implies \neg P$
- contradiction
- induction
- case analysis

*Proof.* Define  $f_{ij}$  as in (5). As  $f$  is symmetric, we only need to consider  $f_{12}$ .

$$\begin{aligned} \mathbf{E} [f_{12}^2] &= \mathbf{E}_{x_3 \dots x_n} \left[ \frac{1}{4} \cdot (f_{12}^2(00x_3 \dots x_n) + f_{12}^2(01x_3 \dots x_n) + f_{12}^2(10x_3 \dots x_n) + f_{12}^2(11x_3 \dots x_n)) \right] \\ &= \frac{1}{4} \mathbf{E}_{x_3 \dots x_n} \left[ (f(00x_3 \dots x_n) - f(11x_3 \dots x_n))^2 + (f(11x_3 \dots x_n) - f(00x_3 \dots x_n))^2 \right] \\ &\geq \frac{1}{2} \left( \binom{n-2}{r_0-1} \cdot 2^{-(n-2)} \cdot 4 + \binom{n-2}{n-r_1-1} \cdot 2^{-(n-2)} \cdot 4 \right) \\ &= 8 \cdot \left( \frac{(n-r_0+1)(n-r_0)}{n(n-1)} \cdot \binom{n}{r_0-1} + \frac{(n-r_1+1)(n-r_1)}{n(n-1)} \cdot \binom{n}{r_1-1} \right) 2^{-n}. \end{aligned}$$

Inequality (6) follows by applying Lemma 2.2.

In order to establish inequality (7), we show a lower bound on the principal Fourier coefficient of  $f$ :

$$\widehat{f}(\emptyset) \geq 1 - 2 \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s > n-r_1} \binom{n}{s} \right) 2^{-n},$$

which implies that

$$\widehat{f}(\emptyset)^2 \geq 1 - 4 \cdot \left( \sum_{s < r_0} \binom{n}{s} + \sum_{s < r_1} \binom{n}{s} \right) 2^{-n}.$$

□

1. What is a proof ?
2. How do you find a proof ?
3. How do you write a proof ?

# How do you write a proof?

<http://www.cs.cmu.edu/~15251/docs/proof-checklist.pdf>

## **PART 2**

Course structure and how to succeed in 15-251

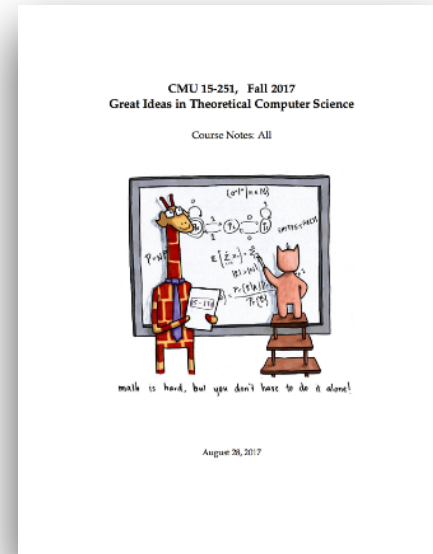
<http://www.cs.cmu.edu/~15251/docs/how-to-succeed.pdf>

# Understand the course structure

## 1. Lecture



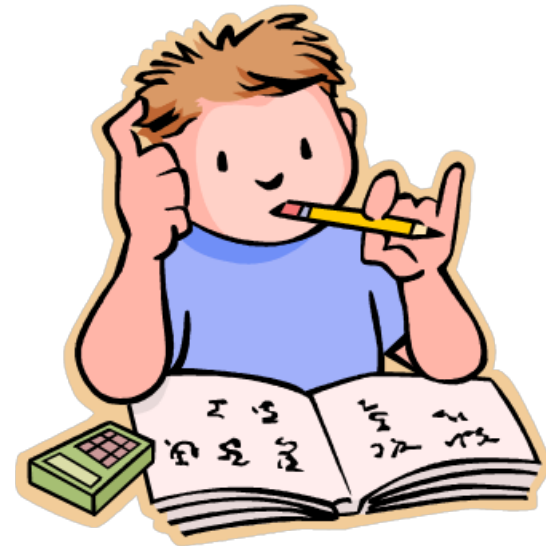
## 2. Course notes



## 3. Recitation



## 4. Homework



# Understand the course structure

## I. Lecture

- provides background, motivation, insights, high-level picture.
- does **not** provide all the details.
- focus in lecture. take notes.

# Understand the course structure

## 2. Course notes

- does **not** provide background and motivation.
- provides the details at the level you need to know them.
- fully understanding concepts and definitions is crucial!!



# Understand the course structure

## 3. Recitation

- basically a small group review session.
- you'll be assigned a 50-minute time slot.
- you'll choose a spiciness level.



- come prepared.

# Understand the course structure

## 4. Homework

- engagement with the material → real learning

# Understand the course structure

## 4. Homework

### **4 types of questions:**

SOLO, GROUP, OPEN COLLABORATION,  
PROGRAMMING

SOLO - work by yourself

GROUP - work in groups of 3 or 4

OPEN - work with anyone you would like from class

PROG - same rules as SOLO. submit to Autolab.

# Understand the course structure

## 4. Homework

Homework comes out Thu night and contains:

SOLO + PROG problems from current week

+

GROUP + OPEN problems from previous week

# Understand the course structure

## 4. Homework

### **Homework writing sessions:**

Wednesdays 6:30pm to 7:50pm at DH 2210

Write the solutions to a random subset of the problems.  
(usually 3 problems)

Practice writing the solutions beforehand!!!

Style matters!!!

# Understand the course structure

## 4. Homework

### **Homework resubmission sessions:**

SIO assigned recitation time and room.

# Understand the course structure

## 4. Homework

### **Homework Grading:**

Satisfactory: We are happy even though there might be some minor errors.

Close: General idea is correct, overall structure is good. But (a) an important piece missing/incorrect, or (b) poor presentation.

Unsatisfactory: The whole thing needs to be rewritten. (up to 2 problems can be rewritten in a resubmission session.)

# Understand the course structure

## 4. Homework

### Homework Grading:

**10pts:** Satisfactory on first submission

**9pts:** Close on first submission, fixes are sent by email.

**8pts:** Close on first submission, no fixes.

**7pts:** Unsatisfactory on first submission, satisfactory on second submission.

**6pts:** Unsatisfactory on first submission, close on second submission.

**0pts:** Rest.



# Find the right group

Your group is going to be one of the most important parts of the course!

# **ADVICE FROM PREVIOUS 15-251 STUDENTS**

*If you leave enough time for 25 I work, it won't be stressful, it'll just be fun. But you have to leave yourself a good amount of time.*

*Be proactive and don't procrastinate! Take advantage of office hours!*

*Go to office hours. They are helpful.*

*get ur shit together and don't be afraid to ask for help.*

***GO TO THE PROF'S OFFICE HOURS AT THE BEGINNING OF  
THE SEMESTER.***

*Read the notes and slides until you completely understand them, then understand the questions on the homework completely before trying to come up with an answer.*

*Understand course material before starting doing homework.  
Definitions are really really important for this class*

*Pay attention in class, go to recitation, review the material every week, and go to office hours.*

*Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls Pls  
practice writing up your proofs before the homework writing  
sessions. I saw a solid letter grade difference whenever I did.*

*Choose your group carefully; make sure that you feel comfortable calling your group members lazy bums if necessary.*

*Find a good group, and expect to be spending a lot of time with them. A lot of the success or failure in the class will come from how well you can work together with your group so that during homework sessions you can all learn something. There will absolutely be problems or concepts which you don't understand as well as someone else in your group, and vice versa. That way you can teach each other, which is ideal. Also, if you get stumped, absolutely attend office hours. The TA's are generally quite helpful.*

*Think of it as a course that will give you a fantastic overview of CS theory – the ride will be tough, but try to focus less on the grades and more on enjoying understanding the material.*





LIFE PROTIP:  
IN THEORY  
EVERYTHING IS  
FINE.