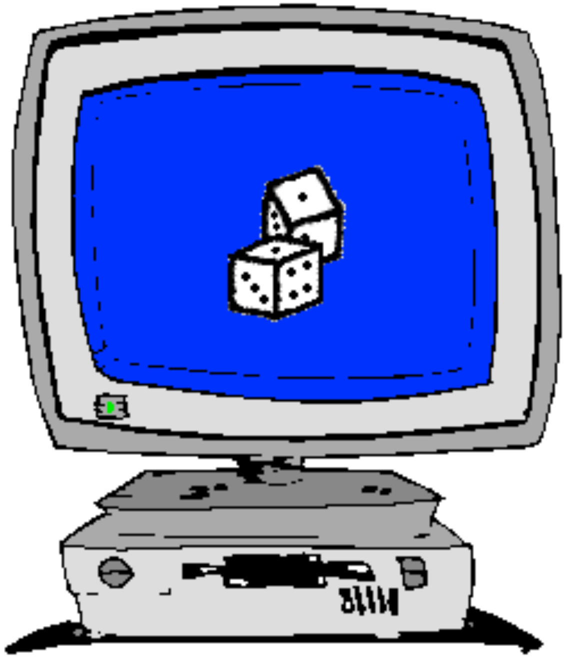
SAMS
Programming - Section C

Week 6 - Lecture 1:
Monte-Carlo method



August 7, 2017

Origins of Probability

France, 1654

Let's bet:

I will roll a dice four times.
I win if I get a 1.

"Chevalier de Méré"

Antoine Gombaud

Origins of Probability

France, 1654

Hmm.

No one wants to take this bet anymore.

“Chevalier de Méré”

Antoine Gombaud

Origins of Probability

France, 1654

New bet:

I will roll two dice, 24 times.

I win if I get double-1's.

"Chevalier de Méré"

Antoine Gombaud

Origins of Probability

France, 1654

Hmm.

I keep losing money!

"Chevalier de Méré"

Antoine Gombaud

Origins of Probability

France, 1654

"Chevalier de Méré"

Antoine Gombaud

Alice and Bob are flipping a coin.

Alice gets a point for heads.

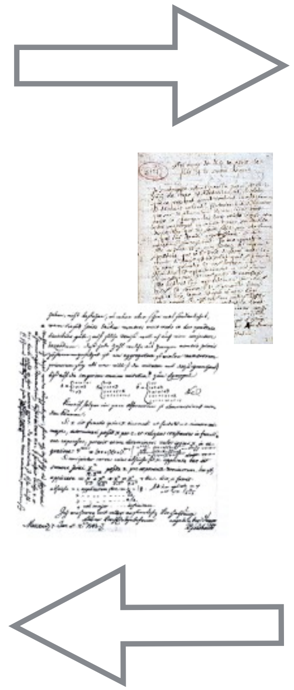
Bob gets a point for tails.

First one to 4 points wins 100 francs.

Alice is ahead 3-2 when gendarmes arrive to break up the game.

How should they divide the stakes?

Origins of Probability



Pascal

Fermat

Probability Theory is born!

Monte Carlo Method

Estimating a quantity of interest (e.g. a probability) by simulating random experiments/trials.

General approach:

Run **trials**

In each **trial**, simulate event (e.g. coin toss, dice roll, etc)

Count **# successful trials**

$$\text{Estimate for probability} = \frac{\#\ \textbf{successful trials}}{\#\ \textbf{trials}}$$

Law of Large Numbers:

As trials —> infinity, estimate —> true probability

Odds of Méré winning

```
def mereOdds():
    trials = 100*1000
    successes = 0
    for trial in range(trials):
        if(mereWins()):
            successes += 1
    return successes/trials


def mereWins():
    for i in range(4):
        dieValue = random.randint(1,6)
        if(dieValue == 1): return True
    return False
```

Example 2: Birthday problem

- Let n = # people in a room.
- Assume people have random birthdays (discard the year).
- What is the minimum n such that:

  Pr[any 2 people share a birthday] > 0.5

(ignore Feb 29)

What is the probability if n = 366?

What is the probability if n = 1?

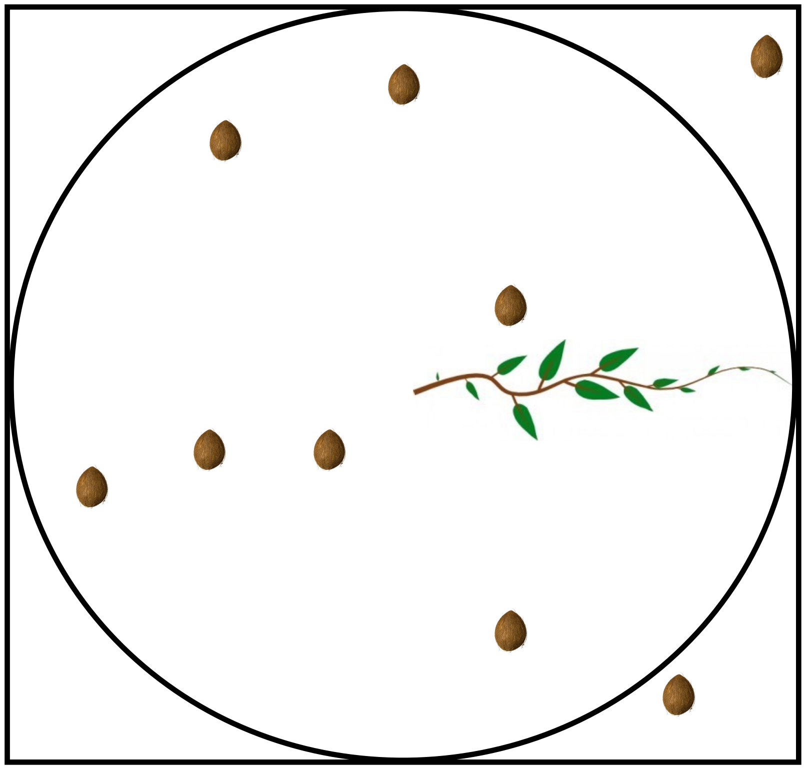
Example 2: Birthday problem

```
def birthdayOdds(n):
    trials = 10*1000
    successes = 0
    for trial in range(trials):
        if trialSucceeds(n):
            successes += 1
    return successes / trials
```

```
def trialSucceeds(n):
    seenBirthdays = “”
    for person in range(n):
        birthday = “$” + str(random.randint(1, 365)) + “$”
        if (birthday in seenBirthdays): return True
        else: seenBirthdays += birthday
    return False
```

Example 3: Estimating Pi

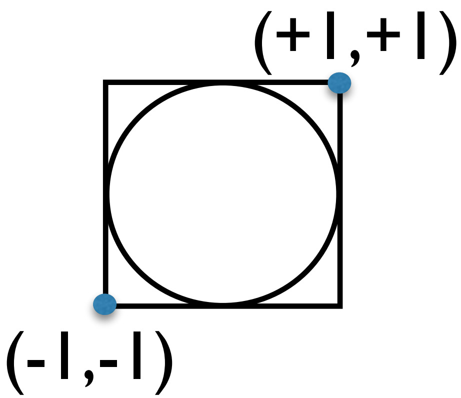
Example 3: Estimating Pi



Pr [random coconut lands in circle] =

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Example 3: Estimating Pi




```
def findPi(throws):            # throws = # trials
    throwsInCircle = 0         # throwsInCircle = # successes
    for throw in range(throws):
        x = random.uniform(-1, +1)
        y = random.uniform(-1, +1)
        if (inUnitCircle(x,y)):
            throwsInCircle += 1
    return 4*(throwsInCircle/throws)
```

```
def inUnitCircle(x,y):
    return (x**2 + y**2 <= 1)
```

Example 4: Monty Hall problem