SAMS
Programming - Section C

Week 6 - Lecture 1:
Monte-Carlo method
Origins of Probability

France, 1654

Let’s bet:
I will roll a dice four times.
I win if I get a 1.

“Chevalier de Méré”
Antoine Gombaud
Origins of Probability

France, 1654

Hmm.

No one wants to take this bet anymore.

“Chevalier de Méré”

Antoine Gombaud
Origins of Probability

France, 1654

New bet:
I will roll two dice, 24 times.
I win if I get double-1’s.

“Chevalier de Méré”
Antoine Gombaud
Origins of Probability

France, 1654

Hmm.
I keep losing money!

“Chevalier de Méré”
Antoine Gombaud
“Chevalier de Méré”
Antoine Gombaud

Alice and Bob are flipping a coin. Alice gets a point for heads. Bob gets a point for tails. First one to 4 points wins 100 francs.

Alice is ahead 3-2 when gendarmes arrive to break up the game. How should they divide the stakes?
Origins of Probability

Pascal

Fermat

Probability Theory is born!
Monte Carlo Method

Estimating a quantity of interest (e.g. a probability) by simulating random experiments/trials.

**General approach:**

Run **trials**

In each **trial**, simulate event (e.g. coin toss, dice roll, etc)

Count **# successful trials**

Estimate for probability = \[
\frac{\text{# successful trials}}{\text{# trials}}
\]

**Law of Large Numbers:**

As **trials** \(\rightarrow\) infinity, **estimate** \(\rightarrow\) true probability
def mereOdds():
    trials = 100*1000
    successes = 0
    for trial in range(trials):
        if(mereWins()):
            successes += 1
    return successes/trials

def mereWins():
    for i in range(4):
        dieValue = random.randint(1,6)
        if(dieValue == 1): return True
    return False
Example 2: Birthday problem

- Let \( n = \# \) people in a room.
- Assume people have random birthdays (discard the year).
- What is the minimum \( n \) such that:
  \[
  \Pr[ \text{any 2 people share a birthday } ] > 0.5
  \]
  (ignore Feb 29)

What is the probability if \( n = 366 \)?

What is the probability if \( n = 1 \)?
Example 2: Birthday problem

```python
def birthdayOdds(n):
    trials = 10*1000
    successes = 0
    for trial in range(trials):
        if trialSucceeds(n):
            successes += 1
    return successes / trials

def trialSucceeds(n):
    seenBirthdays = ""
    for person in range(n):
        birthday = "\$" + str(random.randint(1, 365)) + "\$
        if (birthday in seenBirthdays): return True
        else: seenBirthdays += birthday
    return False
```
Example 3: Estimating Pi
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Pr [ random coconut lands in circle ] =

\[
\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}
\]
Example 3: Estimating Pi

```
def findPi(throws):
    throwsInCircle = 0
    for throw in range(throws):
        x = random.uniform(-1, +1)
        y = random.uniform(-1, +1)
        if (inUnitCircle(x,y)):
            throwsInCircle += 1
    return 4*(throwsInCircle/throws)

def inUnitCircle(x,y):
    return (x**2 + y**2 <= 1)
```
Example 4: Monty Hall problem